1. Show that for every $g \in GL(n,\mathbb{R}) \setminus O(n)$, i.e. $g \in GL(n,\mathbb{R})$ such that $g^T g \neq \mathbb{1}$, there is an open set $U_g$ containing $g$ such that $U_g$ is entirely contained in $GL(n,\mathbb{R}) \setminus O(n)$.

   hint: $GL(n,\mathbb{R})$ inherits its topology from the vector space $V_{n\times n}$ of real $n \times n$ matrices, which is isomorphic to $\mathbb{R}^{n^2}$: the $n^2$ entries of such a matrix are just the components of a vector in $\mathbb{R}^{n^2}$ from this perspective. We can hence describe the open ball of radius $r$ around a matrix $M$ with components $M_{ij}$ as

$$B_r(M) = \left\{ N \in V_{n\times n} | \sum_{ij} (N_{ij} - M_{ij})^2 < r \right\} . \tag{0.1}$$

2. $GL(n,\mathbb{C})$ is the group of invertible complex $n \times n$ matrices. Show that $GL(n,\mathbb{C})$ is a Lie group.

3. Find the dimension of the group $SO(n)$ by finding the dimension of its Lie algebra.

Here are some things to ponder:

1. What are Lie groups?

2. What are Lie algebras?