1. Let $\mathbf{q} \in \mathbb{C}^n$ be acted on in the fundamental representation of $SU(n)$ and γ in the adjoint representation of $SU(n)$ (this is often expressed as $\mathbf{q}$ 'lives' in the fundamental and γ 'lives' in the adjoint of $SU(n)$.)

   By acting with $SU(n)$ simultaneously on γ and $\boldsymbol{q}$, describe the action of $SU(n)$ on

   i) $\mathbf{v} = \gamma \mathbf{q}$

   ii) $\bar{\mathbf{q}}$

   iii) A matrix Q with components $Q_{ij} = q_i q_j$

   and decide in each case if this defines a representation.

2. Consider the map $r_\kappa : U(1) \to GL(3, \mathbb{C})$ defined by

$$r_\kappa(e^{i\phi}) = e^{\phi\lambda\kappa}$$

   where $\kappa \in \mathbb{C}$ and

$$\lambda = \begin{pmatrix} 0 & i & 0 \\ i & 0 & i \\ 0 & i & 0 \end{pmatrix}$$

   For which values of κ is r_κ a representation of $U(1)$? [hint: think about what happens to eigenvectors of λ and use the classification theorem for complex representations of $U(1)$.]

3. Let G be a Lie group and H be a subgroup of G that is also a Lie group.

   a) Explain why any representation $r(G)$ of G also gives us a representation $r(H)$ of H.

   b) Let's assume $r(G)$ is irreducible. Can you think of an example where the representation $r(H)$ is reducible? Can you think of an example where the representation $r(H)$ is irreducible?