26) Show that

$$i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu] \tag{1.1}$$

27) By considering infinitesimal gauge transformations ($|\alpha^a| \ll 1$)

$$g = e^{i\alpha^a t_a} \equiv e^{i\alpha} = 1 + i\alpha + O(\alpha^2) \tag{1.2}$$

and Taylor expanding finite gauge transformations to leading order in $\alpha \in \mathfrak{g} = \mathrm{Lie}(G)$, show that the **infinitesimal gauge variations** of the fields are

$$\begin{aligned} \delta_\alpha \phi &= i\alpha\phi \\ \delta_\alpha A_\mu &= i[\alpha, A_\mu] + \partial_\mu \alpha \\ \delta_\alpha F_{\mu\nu} &= i[\alpha, F_{\mu\nu}] \ , \end{aligned} \tag{1.3}$$

where $\phi \mapsto \phi + \delta_\alpha \phi$ and so on to leading order.

29) Consider a gauge group G, with Lie algebra $\mathfrak{g}$.
  (a) Show by explicit calculation that a non-abelian gauge field configuration of the form

$$A_\mu = ih(\partial_\mu h^{-1}) \ ,$$

  where $h(x)$ is a space-time dependent element of G, has field strength $F_{\mu\nu} = 0$.
  (b) Can you think of a simpler argument to reach the same conclusion?

Here are some things to ponder:

1. How are convariant derivative and field strength defined for a non-abelian gauge theory?
2. What is the impact of charged matter in different representations?