The fate of higher form symmetries in string compactification

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Contents

1. Geometric Engineering of 6D Superconformal Theories (SCFTs)
2. Higher Form Symmetries
3. Coupling to Gravity: Compact Models

6D SCFTs

[Nahm '78]: 6 is highest dimension for SCFTs.

[Witten '95]: type IIB on $\mathbb{C}^2/\Gamma \times \mathbf{R}^{1,5}$ gives 6D SCFT with $N = (2,0)$ if Γ is a finite subgroup of $SU(2) \to$ ADE classification:

Γ	Λ_{ADE}	$\mathfrak{g}$
$\mathbb{Z}_n$	A_{n-1}	$\mathfrak{su}(n)$
Binary Dihedral	D_n	$\mathfrak{so}(2n)$
Binary Tetrahedral	E_6	$\mathfrak{e}_6$
Binary Octahedral	E_7	$\mathfrak{e}_7$
Binary Icosahedral	E_8	$\mathfrak{e}_8$

6D SCFTs: (2,0)

[Witten '95]: type IIB on $\mathbb{C}^2/\Gamma \times \mathbf{R}^{1,5}$ gives 6D SCFT with $N = (2,0)$

Resolution/Deformation gives ALE space X_{ADE}, $H^2(X_{ADE}, \mathbb{Z}) = -\Lambda_{ADE}$.

D3-branes and C_4 on X_{ADE} give rise to BPS strings and self-dual tensor fields $B_a^{(2)}$

Conformal limit: strings become tensionless ... hallmark of 6D SCFTs

Putting these on T^2 gives $N = 4$ SYM, $B_a^{(2)}$ become vectors, $\tau(T^2)$ becomes coupling, strings become dyons, ...

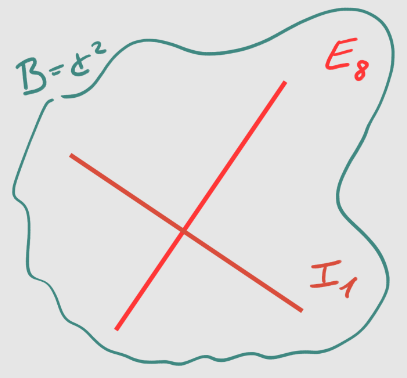
Putting these on Riemannn surfaces gives class S theories

6D SCFTs: (1,0)

Can engineer via F-Theory on non-compact singular elliptically fibred Calabi-Yau threefolds X (IIB with varying dilaton on B)

$$E \to X \to B \qquad y^2 = x^3 + f(z)xw^4 + g(z)w^6 \qquad \Delta = 4f^3 + 27g^2$$

Classic example:
the 'E-string' SCFT [Ganor, Hanany '96]



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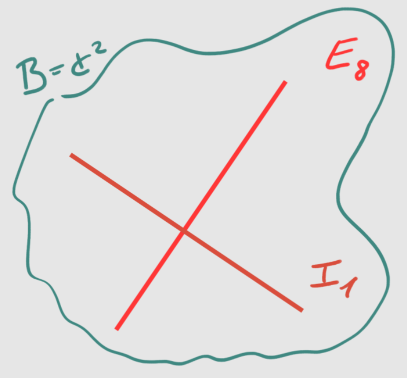

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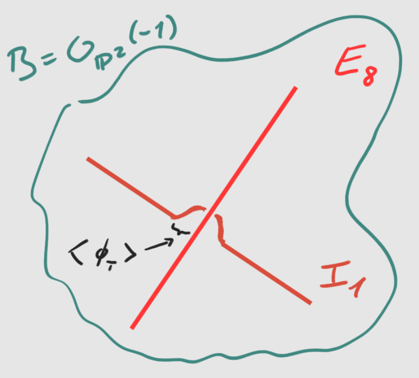
crepant resolution = tensor branch
global (flavour) symmetry = E_8



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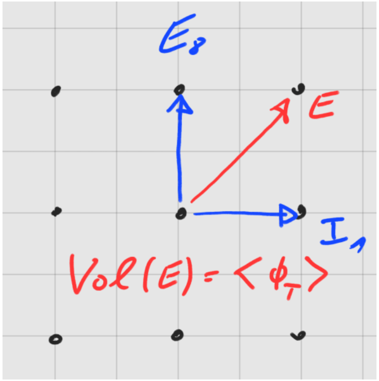
Classic example:
the 'E-string' SCFT [Ganor, Hanany '96]

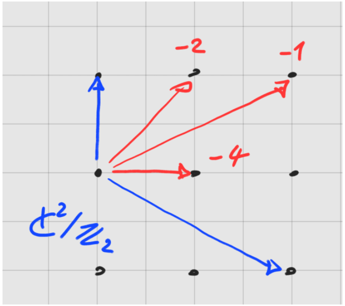
crepant resolution = tensor branch
global (flavour) symmetry = E_8
dual: M5 probing E_8 brane in het. M-theory
het. small instanton

For $\langle \phi_T \rangle = 0$ get tensionless string



6D SCFTs: (1,0)

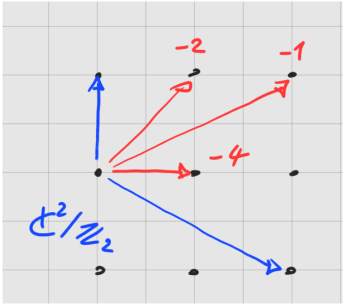
More general: start from F-Theory on smooth base and blow down to $\mathbb{C}^2/\Gamma$ for $\Gamma \in U(2)$ reached [Heckman, Morrison, Vafa '14]



Classification:
[Del Zotto, Heckman, Morrison, Park, Rudelius, Tomasiello, Vafa '13-'15];
caveat: 'frozen' phase not included [Bhardwaj, Morrison, Tachikawa, Tomasiello '18]

6D SCFTs: (1,0)

Lattice of BPS strings Λ_S = lattice generated by curves blown down in B



e.g. $\mathbb{Z}^3$ with inner form $\begin{pmatrix} -2 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -4 \end{pmatrix}$

higher form global symmetries

[Gaiotto, Kapustin, Nathan Seiberg, Willett '15]

6D SCFTs have a 'defect group' of 2-form symmetries G_S [Del Zotto, Heckman, Park, Rudelius '15; Bhardwaj, Schäfer-Nameki '20] acting on 'Wilson surfaces' with representations labelled by $C_a \in H_2(B, \mathbb{Z})$.

$$V(C_a) = \exp\left(i \sum_{ij} a_i \Omega_{ij} \int_{C_a} B_j \right)$$

They transform as

$$V(C_a) \to U_g(S^3) V(C_a) = \exp\left(i \sum_{j} c_j \int_{S_3} H_j \right) V(C_a) = \exp\left(2\pi i \sum_{ij} c_i \, \Omega_{ij} \, a_j \right) V(C_a)$$

defect group

$$V(C_a) \to \exp\left(2\pi i \sum_{ij} c_i\, \Omega_{ij}\, a_j\right) V(C_a) = \exp\left(2\pi i \; c \cdot a\right)$$

The dynamical tensionless strings in $\Lambda_S \subset H_2(B, \mathbb{Z})$ screen the charge of this global symmetry and the only surving group elements are those c such that $c \cdot a \in \mathbb{Z}$. Hence (by definition) $c \in \Lambda_S^*$. The non-trivial transformations in G_S only act on defects in Λ_S^* and

$$G_S = \Lambda_S^* / \Lambda_S \qquad \text{'discriminant group'}$$

A defect $c' \in H_2(B, \mathbb{Z})$ (= string of infinite mass) transforms with the phase

$$\sum_{ij} c_i\, \Omega_{ij}\, c'_j \qquad \text{'discriminant form'}$$

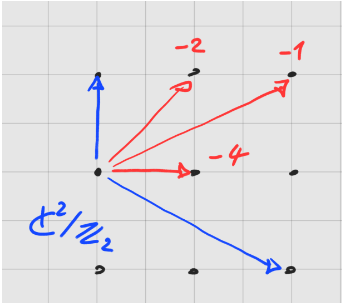
Examples of G_S

For $(2,0)$ theories:

Λ_S	Λ_S^*/Λ_S
A_{n-1}	$\mathbb{Z}_n$
D_{2n}	$\mathbb{Z}_2 \times \mathbb{Z}_2$
D_{2n+1}	$\mathbb{Z}_4$
E_6	$\mathbb{Z}_3$
E_7	$\mathbb{Z}_2$
E_8	-

For our example $(1,0)$ theory:

$$\begin{pmatrix} -2 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -4 \end{pmatrix}$$



$$\Lambda_S^*/\Lambda_S = \mathbb{Z}_2 = \langle (1/2, 0, 1/2) \rangle$$

6D Supergravity

Choosing a compact B (in the $(1,0)$ case) or a K3 surface (in the $(2,0)$ case), we have

$$\Lambda^*/\Lambda = H^2(B,\mathbb{Z})^*/H^2(B,\mathbb{Z}) = 1$$

Hence there is no 2-form symmetry.

The self-duality of $H^2(B,\mathbb{Z})$ is a consequence of Poincaré duality, which fits the self-duality of the charge lattice of 6D supergravity forced by consistency [Kumar, Morrison, Taylor '10; Seiberg, Taylor '11].

The absence of 2-form symmetries is expected from the absence of global symmetries in theories of quantum gravities. [Harlow, Ooguri '18].

What if we choose a compact base and track the behavior of the two-form symmetries ? $\mathcal{N} = (2,0)$

Moduli space of IIB on a K3 surface: Grassmanian of 5-planes Σ_5:

$$O(\Lambda_{5,21}) \setminus O(5,21) / \left(O(5) \times O(21)\right) \ .$$

If $\Sigma_5 \perp \oplus_i \Gamma_i = \Lambda_S$ the theory has the associated SCFTs as subsectors which have 2-form symmetries

$$G_S = \Lambda_S^* / \Lambda_S = \oplus_i \Gamma_i^* / \Gamma_i$$

However, now we should think of all elements of $\Lambda_{5,21} = U^5 \oplus (-E_8)^2$ as dynamical objects screening 2-form symmetries. Unscreened subgroup G of G_S must act trivially on $\eta \in \Lambda_{5,21}$, i.e. need γ s.t. $\gamma \cdot \eta \in \mathbb{Z}$ for all $\eta \in \Lambda_{5,21}$ Hence $\gamma \in \Lambda_{5,21}^* = \Lambda_{5,21}$ and

$$G = (\Lambda_{5,21} \cap \Lambda_S^*) / \Lambda_S$$

primitivity

An embedding $\Lambda_S \hookrightarrow \Lambda_{5,21}$ is called primitive if

$$\Lambda_{5,21} \cap (\Lambda_S \otimes \mathbb{Q}) = \Lambda_S$$

primitive

not primitive

primitivity and G

An embedding $\Lambda_S \hookrightarrow \Lambda_{5,21}$ is called primitive if

$$\Lambda_{5,21} \cap (\Lambda_S \otimes \mathbb{Q}) = \Lambda_S$$

Rewrite

$$G = (\Lambda_{5,21} \cap \Lambda_S^*)/\Lambda_S = \Lambda_{5,21} \cap (\Lambda_S \otimes \mathbb{Q})\,/\Lambda_S$$

Hence if $\Lambda_S \hookrightarrow \Lambda_{5,21}$ is primitive, G is trivial.

In general:

$$G = \text{tor}\ \ (\Lambda_{5,21}/\Lambda_S)$$

tor: elements η of $\Lambda_{5,21}$ that are not in Λ_S, but a multiple of η is in Λ_S.

$G = \text{tor}\ (\Lambda_{5,21}/\Lambda_S)$

By construction, each Γ_i is primitively embedded into $\Lambda_{5,21}$ but together they do not need to be. Example:

$$E_8 = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \qquad \text{all } a_i \in \mathbb{Z} \text{ or } \mathbb{Z} + \tfrac{1}{2} \qquad \sum a_i \in 2\mathbb{Z}$$

Sublattice A_1^4 generated by:

$$\begin{aligned} \eta_1 &= (1,1,0,0,0,0,0,0) \\ \eta_2 &= (0,0,1,1,0,0,0,0) \\ \eta_3 &= (0,0,0,0,1,1,0,0) \\ \eta_4 &= (0,0,0,0,0,0,1,1) \end{aligned}$$

$\eta_1 + \eta_2 + \eta_3 + \eta_4 = (1^8)$. But now $(1/2^8)$ is in $(A_1^4 \otimes \mathbb{Q}) \cap E_8$ but not A_1^4.

$G = \text{tor}\ (\Lambda_{5,21}/\Lambda_S)$

General class of (geometric) models: consider an elliptically fibred K3 surface; Moduli space

$$O(\Lambda_{2,18}) \setminus O(2,18)/\left(O(2) \times O(18)\right) \ .$$

ADE singularities: $\Gamma_i \perp \Sigma_2$. $\Gamma_i \hookrightarrow \Lambda_{2,18}$. $\Lambda_S = \oplus_i \Gamma_i$

$$\text{tor}\ (\Lambda_{5,21}/\Lambda_S) = \text{tor}\ (\Lambda_{2,18}/\Lambda_S) = \text{tor}(\text{Mordell-Weil group})$$

Mordell-Weil group $=$ the group of sections of the fibration. Hence torsional sections give rise to unbroken 2-form symmetries.

But they should not be there by the general logic ! $\rightarrow$ they must be gauged!

This is not unfamiliar from 0-form, i.e. flavour, symmetries.

gauged 2-form symmetries and duality

Consider again putting IIB on an elliptically fibred K3 surface X with certain ADE singularities $\sim$ singular fibres $\sim$ algebras $\mathfrak{g}_i$.
Gauged two-form symmetries:

$$G = \text{tor}\, MW(X)$$

Put this theory on S^1. Gauge group of 5D theory is

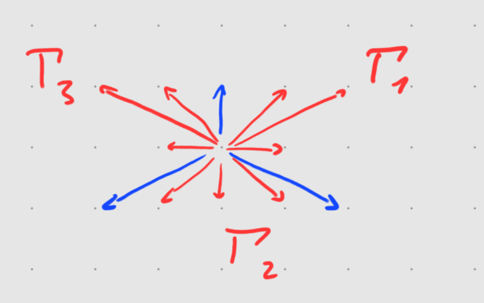
$$\otimes_i G_i / G$$

We can also think of this as F-Theory on $E \times X \times S^1$ or M-Theory on $E \times X$.
This is dual to F-Theory on X (using the elliptic fibration on X) times E times S^1. Here it is known that tor $MW(X)$ changes the global structure of the gauge group as observed above [Mayrhofer,Morrison,Till,Weigand '14]

Heterotic version: het strings on T^5 [Fraiman, Parra De Freitas '21]

$(1, 0)$ theories

Start from a smooth 3fold with base B and blow down $B \to B_o$



The structure of how the (not self-dual) lattice Λ_S sits inside the self-dual lattice $H_2(B, \mathbb{Z})$ was answered by [Del Zotto, Heckman, Morrison, Park '14].

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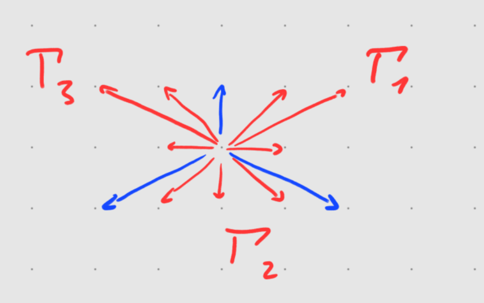

Again

$$G = \left(H_2(B, \mathbb{Z}) \cap \Lambda_S^*\right) / \Lambda_S$$

where $\Lambda_S = \oplus_i \Gamma_i$.

$(1,0)$ theories

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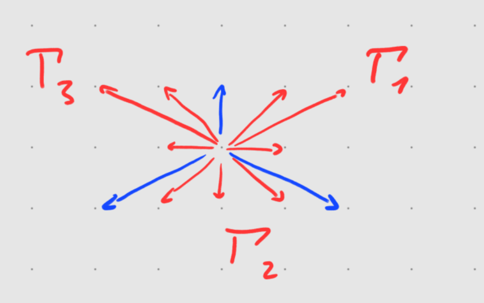

If $B_o = \hat{B}_o/\mathbb{Z}_n$ then

$$G = \mathbb{Z}_n$$

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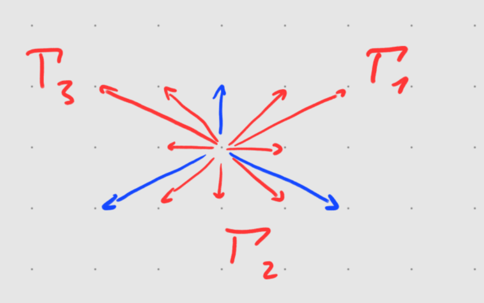

If $B_o = \hat{B}_o/\mathbb{Z}_n$ then

$$G = \mathbb{Z}_n$$

Again: can make examples where this is mapped to torsional Mordell-Weil group under duality

Thank you!

- 6D SCFTs have 2-form symmetries G_S
- These can be subsectors of 6D supergravity theories
- G_S must be broken or gauged
- worked out when there is a gauged subgroup G for $(2,0)$ theories and $(1,0)$ theories engineered in F-Theory with toric bases

general lesson:

$$G = \text{tor}\left(\frac{\text{charge lattice}}{\text{charge lattice of SCFT sector}}\right)$$

similar: gauged 1-form symmetries and global structure of gauge group