Nonparametric predictive reliability of series of voting systems

Frank Coolen

Durham University

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Joint work with Ahmad Aboalkhair and Iain MacPhee

Part of Ahmad's PhD thesis (2012) (available from my webpage)

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The start of this research - 2007

What is the probability that a series system with m 'identical' components functions?

A. If each component functions with probability 0.5.

B. If two such components were tested, one of which functioned and one failed (no further information)

What about a parallel system?

Using NPI for Bernoulli quantities

$$\overline{P}(Y_{n+1}^{n+m} \geq k | Y_1^n = s) = \binom{n+m}{n}^{-1} \times$$

$$\left[\binom{s+k}{s}\binom{n-s+m-k}{n-s} + \sum_{l=k+1}^{m} \binom{s+l-1}{s-1}\binom{n-s+m-l}{n-s} \right]$$

$$\underline{P}(Y_{n+1}^{n+m} \geq k | Y_1^n = s) =$$

$$1 - \binom{n+m}{n}^{-1} \sum_{l=0}^{k-1} \binom{s+l-1}{s-1}\binom{n-s+m-l}{n-s}$$

Can derive via counting 'right-up' paths from $(0,0)$ to (n,m)

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We considered systems such that

- Subsystems in series configuration
- Each subsystem a k^l-out-of-m^l system
- Each subsystem can consist of multiple component types
- Same component types can be in different subsystems
- Test data per type of component (‘exchangeable’)
- Components of different types independent

We derived the NPI upper and lower probabilities for the event that the system functions

(quite horrible expressions..)

i	subsystem i	m_a^i	m_b^i	m_c^i
1	1-out-of-6	2	2	2
2	2-out-of-6	2	2	2
3	3-out-of-6	4	0	2

Test data: 3 components of each type tested, of which functioned:
Type A: 3
Type B: 2
Type C: 1

Extra	m_a^1, m_b^1, m_c^1	m_a^2, m_b^2, m_c^2	m_a^3, m_b^3, m_c^3	$\underline{P}$
0	2,2,2	2,2,2	4,0,2	0.7256
1	2,2,2	2,2,2	5,0,2	0.7896
2	2,2,2	3,2,2	5,0,2	0.8302
3	2,2,2	3,2,2	6,0,2	0.8675
4	2,2,2	3,2,2	6,1,2	0.8907
5	2,2,2	4,2,2	6,1,2	0.9087
6	2,2,2	4,2,2	6,2,2	0.9244
7	2,2,2	4,2,2	7,2,2	0.9345
8	2,2,2	5,2,2	7,2,2	0.9437
9	3,2,2	5,2,2	7,2,2	0.9513
10	3,2,2	5,2,2	7,3,2	0.9584
11	3,2,2	6,2,2	7,3,2	0.9635
12	3,2,2	6,2,2	8,3,2	0.9684
13	4,2,2	6,2,2	8,3,2	0.9716
14	4,2,2	6,2,2	8,4,2	0.9748
15	4,2,2	6,3,2	8,4,2	0.9782
16	4,2,2	6,3,2	9,4,2	0.9806
17	4,2,2	7,3,2	9,4,2	0.9828
18	5,2,2	7,3,2	9,4,2	0.9845

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Main results

- For basic systems, ‘myopic optimal’ redundancy allocation is optimal (papers 2008, 2009)
- For more complicated systems, as above, this is a conjecture
- Continuing adding components, *all* component types with at least one functioning component in tests will be added (eventually)
- Focus on lower probability of system functioning is attractive with regard to risk analysis

Output

Coolen-Schrijner P, Coolen FPA, MacPhee IM (2008). Nonparametric predictive inference for systems reliability with redundancy allocation. *JRR 222* 463-476.

MacPhee IM, Coolen FPA, Aboalkhair AM (2009). Nonparametric predictive system reliability with redundancy allocation following component testing. *JRR 223* 181-188.

Aboalkhair AM, Coolen FPA, MacPhee IM. Nonparametric predictive inference for reliability of a voting system with multiple component types. *RESS*, to appear in 2014 (!).

Aboalkhair AM, Coolen FPA, MacPhee IM (2013). Nonparametric predictive reliability of series of voting systems. *EJOR 226* 77-84.

and several further publications (conference proceedings, edited volume, magazine)

Durham University

Ahmad Aboalkhair

Durham University

Ahmad’s words

Dr. Iain MacPhee was the first person who I met at the Department of Mathematical Sciences when I arrived in Durham at the start of May 2008.

He was instrumental in giving my thoughts the right direction, and then sadly passed away in January 2012 following a long standing battle with cancer before he could see the product of his guidance.

He was an excellent supervisor. He patiently taught me so much that has enriched my knowledge, and I wish I could tell him how much I appreciate that.

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