Server Advantage in Tennis Matches

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SERVER ADVANTAGE IN TENNIS MATCHES

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Abstract

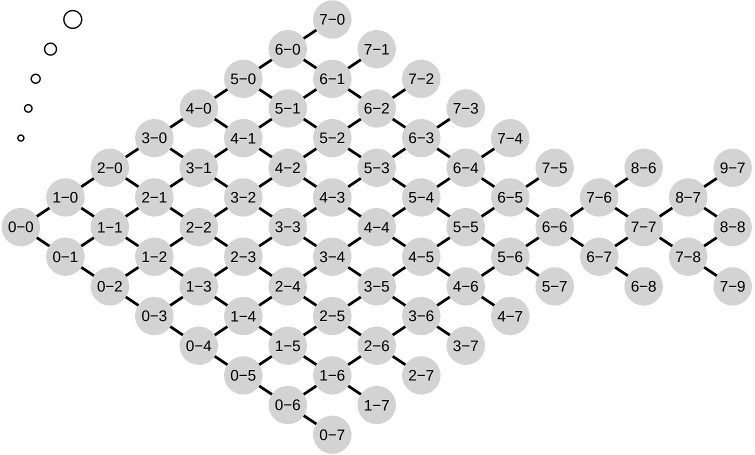
We show that the advantage that can accrue to the server in tennis does not necessarily imply that serving first changes the probability of winning the match. We demonstrate that the outcome of tie-breaks, sets and matches can be independent of who serves first. These are corollaries of a more general invariance result that we prove for n-point win-by-2 games. Our proofs are non-algebraic and self-contained.

Keywords: Tennis; tie-break; n-point win-by-k games

2000 Mathematics Subject Classification: Primary 91A60; 91A05
Secondary 60J20

The fish plot

A tennis tie-break is an example of a 7-point win-by-2 game.



etc. etc. on the righthand side.

The Flip-Flop Lemma

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Individual points comprise independent Bernoulli trials with fixed probability of success depending only on a single two-level factor, say $\{A, B\}$.

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Flip-Flop Lemma

Let the factor levels conform to a PFO with given rule. Under the assumption, the probability of attaining the terminal score i-j is invariant to the rule when either

(i) the game is played for exactly $2m$ points, or

(ii) the game is n-point win-by-2, and $\min\{i, j\} \geq n - 1$.

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	1	0	1	1	0	1	1	0	5–3
Rule R_A	A	B	B	A	A	B	B	A	

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	0	1	1	1	1	0	0	1	5−3
Rule R_B	B	A	A	B	B	A	A	B	

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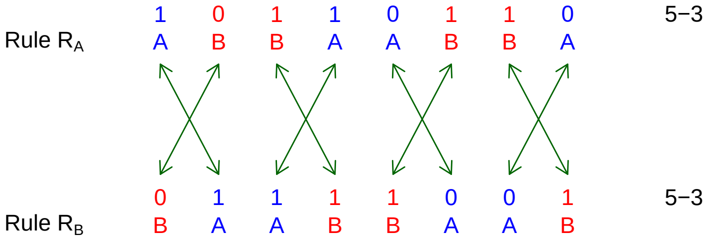

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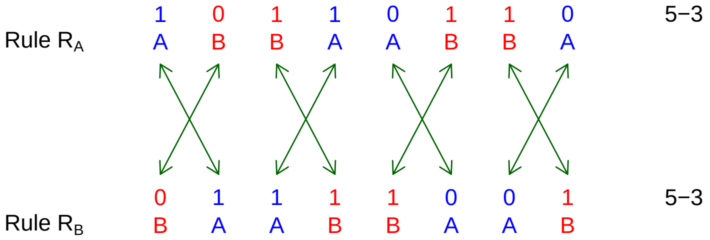

- These two paths to 5-3 have the same probability. The probability of the outcome 5-3 under R_A is the sum of the probabilities of the paths to 5-3. The bijective relationship between paths under R_A and R_B shows that the probability of 5-3 is the same for R_A and R_B.

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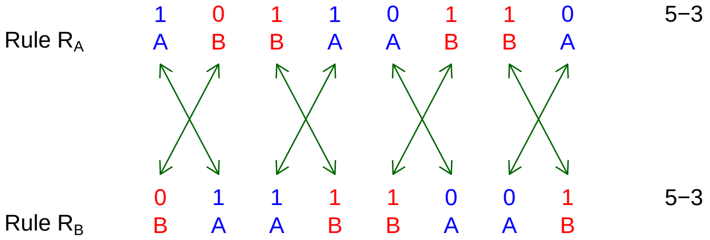

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- The conditions of the Lemma ensure that there are an even number of points, and that the swapping operation does not imply a different terminating score.

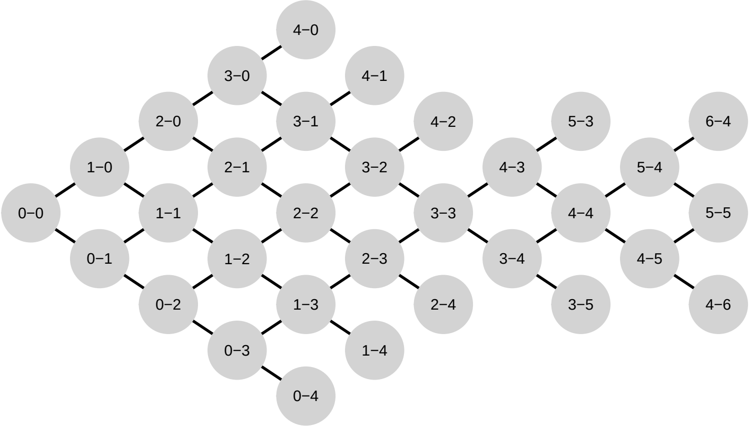
Theorem

Under the assumption, the probability that A wins a tiebreak does not depend on who serves first.

Corollaries

The same result also holds for sets and for matches.

Proof of the theorem (fish plot is $n = 4$)



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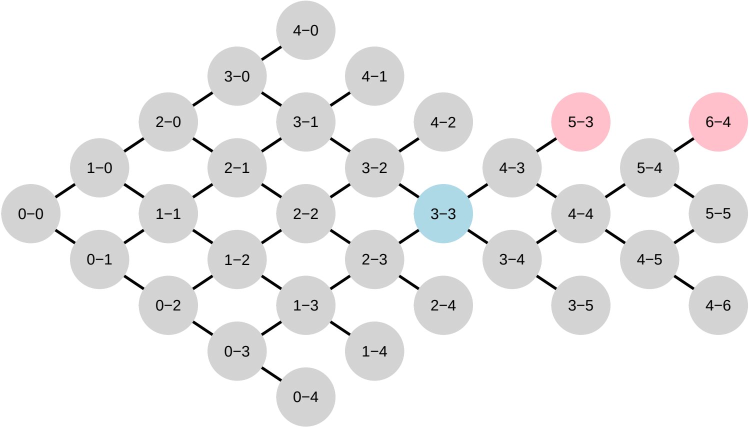
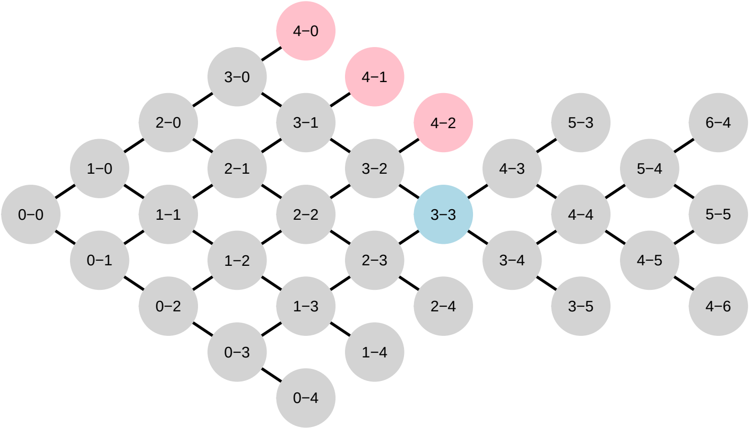


We condition on the path passing through $(n-1)$-$(n-1)$. If this point is on the path, then all terminating points satisfy case (ii) of the Flip-Flop Lemma.

Proof of the theorem (fish plot is $n = 4$)



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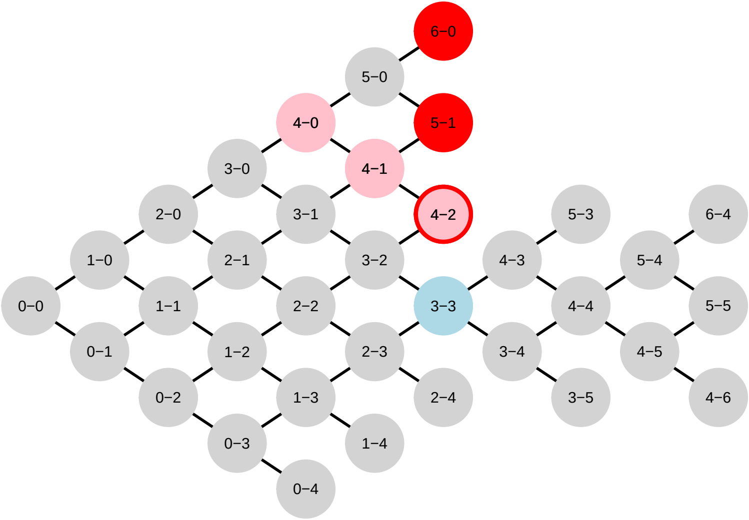

For paths not passing through this point, *the probability on the two sets, pink and red, is the same*. But each of the red points satisfies case (i) of the Flip-Flop Lemma.

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2. Sets *with* tiebreaks. We condition our argument on passing through the score 5-5 in games. If we pass through 5-5, then each of 7-5, 5-7, and the tiebreak are invariant to the PFO. If we do not pass through 5-5, then apply the same reasoning as the second branch of the tiebreak proof.

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So it does not matter, for winning the set, who serves first. And hence it does not matter for winning the match.