Majority Consensus
Algorithms and Spectral Optimisation

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Computational power of (social) networks

Off to the movies...

- Friends who want to watch one of two movies (together)
- They interact in pairs in order to come to a common choice before the end of the day
- Majority prefers one theater

What protocol should they run to decide to go to the latter?

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Biased voting [Kearns et al '09]

- Players paid if consensus conforms with their preference
- For some network topologies, minority preference consistently wins
- Individual behavioral characteristics (stubbornness, awareness of opposing incentives) correlate with earnings

Consensus on networks

Distributed computing

- Information fusion/consistency in distributed networks
- Network awareness (Computing graph properties)
- Multi-agent coordination and flocking

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Distributed algorithms

- Numerous impossibility results in the deterministic case [Lynch et al '90s]
- Random walks
- Gossiping algorithms

Binary majority consensus

Desired outcome and metrics

- Nodes end with opinion held by majority of nodes
- Node can probe neighbours and update opinion accordingly using little (constant) memory
- Probability of error (convergence to incorrect consensus)
- Time to convergence

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Applications

- Occurrence of a given event in cooperative decision making
- Voting in distributed systems
- Routine to solve more elaborate distributed decision making instances

Outline

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Model

Interaction Model

- Connected undirected graph $G = (V, E)$, $|V| = n$
- αn nodes hold 0 and $(1-\alpha)n$ nodes hold 1, $\alpha \in (1/2, 1)$
- Nodes i and j interact at rate $q_{ij} = q_{ji}$, $q_{ij} \neq 0$ iff $(i, j) \in E$

Markov chain

- $(X_t)_{t\geq 0}$ with rate matrix Q, $q_{ii} = -\sum_{i\neq j} q_{ij}$
- $(\pi_i)_{i\in V}$ stationary distribution is uniform on V. Mixing time:

$$\left|\mathbb{P}_j(X_t = i) - 1/n\right| = O\left(e^{-\lambda_2(Q)t}\right)$$

where $\lambda_2(Q) = \inf\{\sum_{i,j} q_{ij}(x_i - x_i)^2, ||x|| = 1,\ x^T\mathbf{1} = 0\}$

Probability of error

Interaction Model

- Connected undirected graph $G = (V, E)$, $|V| = n$
- αn nodes hold 0 and $(1 - \alpha)n$ nodes hold 1, $\alpha \in (1/2, 1)$
- Nodes i and j interact at rate q_{ij} and i updates to j's state w.p. $1/2$

Theorem [Liggett '85, Hassin-Peleg '01]

- The number of nodes in state 1 is a martingale.
- Probability of reaching (wrong) consensus at 1 is $1 - \alpha$.

Time to convergence [Aldous 2012]

Complete graph

- Each edge has rate $1/(n-1)$. The number of agents with opinion 1 evolves as Birth and Death proces with

$$\lambda_{k,k+1} = \lambda_{k,k-1} = \frac{k(n-k)}{2(n-1)} .$$

- Time to convergence $= O(n)$

General graph

- Conductance $\eta(Q) = \inf_{A \subset V} \frac{\sum_{i \in A, j \in A^c} q_{ij}}{|A||A^c|/n}$
- The Markov chain tracking the number of nodes in state 0 evolves at least $\eta(Q)$ times as fast as on the complete graph,
  Time to convergence $O(n/\eta(Q))$

Time to convergence [Cooper et al 2012.]

Cheeger's inequality

- Conductance: $\eta(Q) = \inf_{A \subset V} \frac{\sum_{i \in A, j \in A^c} q_{ij}}{|A||A^c|/n}$
- Spectral Gap:
  $\lambda_2(Q) = \inf\{\sum_{i,j} q_{ij}(x_i - x_j)^2, ||x|| = 1,\ x^T \mathbf{1} = 0\}$

$$\lambda_2(Q) \leq \eta(Q)\ .$$

- Time to convergence of voter model $O(n/(\lambda_2(Q)))$.

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Let S of size k be the subset realising the inf in $\eta(Q)$ and let x such that $x_i = -\sqrt{\frac{n-k}{kn}},\ i \in S$ and $x_i = \sqrt{\frac{k}{(n-k)n}},\ i \in S^c$.

Distributed averaging

At each interaction of (i, j) occuring at rate q_{ij}

$$x_i(t) = x_j(t) = \frac{x_i(t-) + x_j(t-)}{2} .$$

Theorem [Boyd et al '06, Aldous-Lanoue '12]

- Algorithm converges to the average value, using $O(Poly(\log(n))$ memory per node
- Time to convergence to up $O(1/n)$ error of the average is

$$\Theta(\log(n)/\lambda_2(Q)) ,$$

Distributed averaging: Proof

Assume that $\sum_i x_i(0) = 0$.
Let $Q(t) = ||x(t)||^2$. When an i, j interaction takes place $Q(t)$ reduces by $(x_i - x_j)^2/2$.

$$\begin{aligned} \mathbb{E}(dQ(t) \mid x(t) = x) &= -\sum_{i,j} q_{ij} \frac{(x_i - x_j)^2}{2} dt \\ &\leq -\lambda_2(Q)||x||^2/2dt \end{aligned}$$

In particular

$$\mathbb{E}||x(t)||^2 \leq ||x(0)||^2 e^{-\lambda_2(Q)t/2}$$

Small memory

Could we use less memory and still guarantee small error?

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Impossibility [Land-Belew '95]

- Connected undirected graph $G = (V, E)$, $|V| = n$,
- αn nodes in 0 and $(1-\alpha)n$ nodes in 1, $\alpha \in (1/2, 1)$, $2\alpha - 1$ is the *voting margin*.
- i contacts j at rate $q_{ij} > 0 \; \forall (i, j) \in E$

No 1-bit distributed algorithm can solve the majority consensus problem.

Binary Consensus with two undecided states

Averaging-like updates: States $0 < e_0 < e_1 < 1$.

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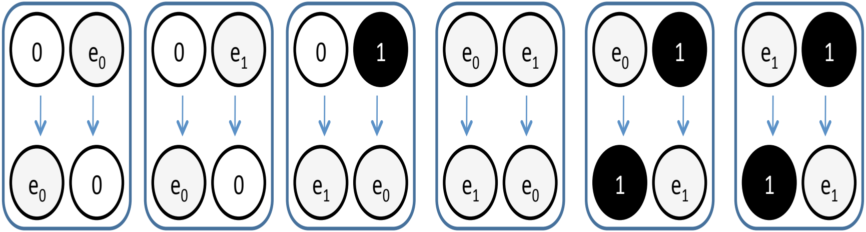
Rules: Swaps + Annihilation



Mean-field analysis (Complete graph)

Let $q_{ij} = \frac{1}{n-1}$, $i \neq j$ and $\mathbf{X}(t) = (|S_0(t)|, |S_{e_0}(t)|, |S_{e_1}(t)|, |S_1(t)|)$ is a Markov process with the following transition rates

$$\rightarrow \begin{cases} (|S_0(t)| - 1, |S_{e_0}(t)| + 1, |S_{e_1}(t)| + 1, |S_1(t)| - 1) & : \frac{|S_0(t)||S_1(t)|}{n-1} \\ (|S_0(t)|, |S_{e_0}(t)| - 1, |S_{e_1}(t)| + 1, |S_1(t)|) & : \frac{|S_{e_0}(t)||S_1(t)|}{n-1} \\ (|S_0(t)|, |S_{e_0}(t)| + 1, |S_{e_1}(t)| - 1, |S_1(t)|) & : \frac{|S_0(t)||S_{e_1}(t)|}{n-1} \end{cases}.$$

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By Kurtz, $\mathbf{X}(t)/n$ converges to $(s_0(t), s_{e_0}(t), s_{e_1}(t), s_1(t))$,

$$s_{e_1}(t) \sim (2\alpha - 1)\frac{1-\alpha}{\alpha} t e^{-(2\alpha-1)t}$$

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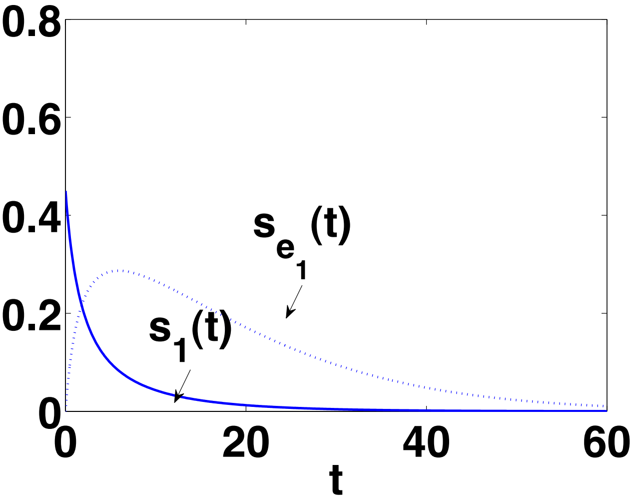
Minority states



General bound

Theorem [Draief-Vojnovic '12]

Let T be the time until there are only nodes in state 0 and e_0.

$$\mathbb{E}(T) = O\left(\log n/\delta(G,\alpha)\right)$$

where

$$\delta(Q,\alpha) = \min_{S\subset V, |S|=(2\alpha-1)n} \min_{\lambda\in Spec(Q_S)} |\lambda|$$

Generalised conductance lemma [Babaee-Draief '13+]

We have

$$\delta(Q,\alpha) \geq c_\alpha \lambda_2(Q)$$

In particular,

$$\mathbb{E}(T) = O(\log(n)/\lambda_2(Q))$$

Proof: Depletion of nodes in state 1

Let A_i and Z_i indicator node in 0 and 1 resp. The transitions of the Markov process (Z, A) is given by

$$(Z, A) \rightarrow \begin{cases} (Z - e_i, A - e_j) & : \quad q_{i,j} Z_i A_j \\ (Z - e_i + e_j, A) & : \quad q_{i,j} Z_i (1 - A_j - Z_j) \\ (Z, A - e_i + e_j) & : \quad q_{i,j} A_i (1 - A_j - Z_j) \end{cases}$$

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For $t \in [t_k, t_{k+1})$ where $\{S_0(t) = S_k\}$

$$\frac{d}{dt}\mathbb{E}_k(A_i(t)) = -\left(\sum_{l\in V} q_{i,l}\right)\mathbb{E}_k(A_i(t)) + \begin{cases} \sum_{j\in V} q_{i,j}\mathbb{E}_k\left(A_j(t)\right), & i \notin S_k \\ 0, & i \in S_k \end{cases}$$

where $\mathbb{E}_k$ is the expectation conditional on $\{S_0(t) = S_k\}$.

(random) Piecewise-linear dynamical system

Dynamics

The dynamics of the system boils down to $Y(t) = (Y_i(t))_{i \in V}$,

$$\frac{d}{dt}\mathbb{E}_k(Y(t)) = Q_{S_k}\mathbb{E}_k(Y(t)) \, ,$$

for $t \in [t_k, t_{k+1})$ during which $\{S_0(t) = S_k\}$ and Q_{S_k} is given by

$$Q_S(i,j) = \begin{cases} -\sum_{l \in V} q_{i,l}, & i = j \\ q_{i,j}, & i \notin S, j \neq i \\ 0, & i \in S, j \neq i. \end{cases}$$

Solution

Proposition

Solving the above differential equation and using the strong Markov property

$$\mathbb{E}(Y(t)) = \mathbb{E}\left[e^{\lambda(t)} Y(0)\right]$$

where $\lambda(t) = Q_{S_k}(t - t_k) + \sum_{l=0}^{k-1} Q_{S_l}(t_{l+1} - t_l)$.

Proof: Spectrum of Q_S

For any finite graph G, there exists $\delta(G, \alpha) > 0$ such that, for any non-empty subset of vertices S with $|S| \in [(2\alpha - 1)n, \alpha n]$, if λ is an eigenvalue of the the matrix Q_S defined in, then

$$\lambda \leq -\delta(G, \alpha) < 0.$$

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- First $\left(-\sum_{l\in V} q_{i,l}\right)$, $i \in S$ are eigenvalues of Q_S
- The remaining eigenvalues correspond to eigenvectors of the form $(\underbrace{x}_{S^c}, \underbrace{0,\ldots,0}_{S})^T$. Let $W \subset S^c$ such that for $i \in W$, $x_i \neq 0$

$$-\lambda = \sum_{i\in W}\sum_{j\in S} q_{i,j}x_i^2 + \sum_{i\in W, j\in S^c\setminus W} q_{i,j}x_i^2 + \frac{1}{2}\sum_{i,j\in W} q_{i,j}(x_i - x_j)^2$$

Proof: The End

Note that

$$\mathbb{E}(Y(t)) = \mathbb{E}\left[e^{\lambda(t)} Y(0)\right]$$

where $\lambda(t) = Q_{S_k}(t - t_k) + \sum_{l=0}^{k-1} Q_{S_l}(t_{l+1} - t_l)$

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Hence

$$||\mathbb{E}(Y(t))||_2 \leq \mathbb{E}\left[||e^{Q_{S_k}(t-t_k)}|| \prod_{l=0}^{k-1} ||e^{Q_{S_l}(t_{l+1}-t_l)}|| \, ||Y(0)||_2\right] \leq \sqrt{n}e^{-\delta(G,\alpha)t}$$

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Therefore, by Cauchy-Schwartz, we have

$$\mathbb{P}(Y(t) \neq \mathbf{0}) \leq \sum_{i \in V} \mathbb{E}(Y_i(t)) \leq n \, e^{-\delta(G,\alpha)t}$$

Summary

- Upper bound on the expected convergence time for a number of distributed candidate dynamics for solving Majority consensus
- Bounds based on the location of the spectral gap of rate matrix (generalised-cut: quick for expander graphs).
- For binary consensus, expected convergence time critically depends on the voting margin

Summary

- Upper bound on the expected convergence time for a number of distributed candidate dynamics for solving Majority consensus

- Bounds based on the location of the spectral gap of rate matrix (generalised-cut: quick for expander graphs).
- For binary consensus, expected convergence time critically depends on the voting margin

- Application to particular network topologies: complete graphs, stars, ER graph, paths, cycles.

Outline

Upper Bounds

Corollary

An application of the theorem to complete graph $q_{i,j} = \frac{1}{n-1}$ for all $i \neq j$, yields

$$\mathbb{E}(T_i) \leq \frac{1}{2\alpha - 1} \log(n).$$

Exact asymptotics

A direct analysis of the dynamics of the 1st phase tracking the interactions of nodes in state 1 and nodes in state 0 implies that

$$\mathbb{E}(T_1) = \frac{n-1}{|S_0| - |S_1|} \left(H_{|S_1|} + H_{|S_0|-|S_1|} - H_{|S_0|} \right)$$

where $H_k = \sum_{i=1}^{k} \frac{1}{i}$

Various initial conditions

- $|S_0| - |S_n| = (2\alpha - 1)n$, α a constant larger than $1/2$

$$\mathbb{E}(T_1) = \frac{1}{2\alpha - 1}\log(n) + O(1).$$

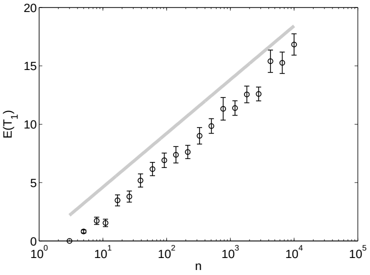
- If $|S_0| = |S_1|$

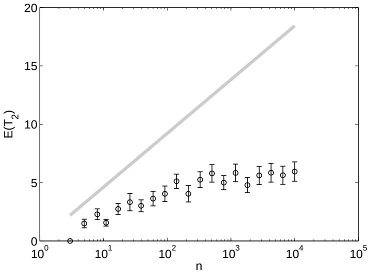
$$\mathbb{E}(T_1) = \frac{\pi^2}{6} n(1 + o(1)).$$

- $\mu_n = (|S_0| - |S_1|)/n$ is strictly positive but small ($o(1)$),

$$\mathbb{E}(T_1) = \frac{1}{\mu_n}\log(n\mu_n) + O(1).$$

Complete Graph: Theory v. Simulation





Star

- **Star Network:** $q_{1,i} = q_{i,1} = \frac{1}{n-1}$, $i \neq 1$ and $q_{i,j} = 0$, $i, j \neq 1$. $\mathbb{E}(T_i) \leq \frac{1}{2\alpha-1} n \log(n)$. Using, direct calculation

$$\mathbb{E}(T_1) = \frac{1}{(2\alpha-1)(3-2\alpha)} n \log(n) + O(n)$$

- **ER-graph:** $q_{i,j} = \frac{1}{np_n} X_{i,j}$ $X_{i,j}$ i.i.d. Bernoulli r.v. with mean $c\frac{\log(n)}{n}$, $c > \frac{2}{2\alpha-1}$, for h^{-1} the inverse of $h(x) = x\log(x) + 1 - x$,

$$\mathbb{E}(T_i) \leq \frac{1}{(2\alpha-1)h^{-1}\left(\frac{2}{c(2\alpha-1)}\right)} \log(n) + O(1)$$

- **Path**: $\mathbb{E}(T_i) \leq \frac{16(1-\alpha)^2}{\pi^2} n^2 \log(n) + O(1)$
- **Ring**: $\mathbb{E}(T_i) \leq \frac{4(1-\alpha)^2}{\pi^2} n^2 \log(n) + O(1)$.

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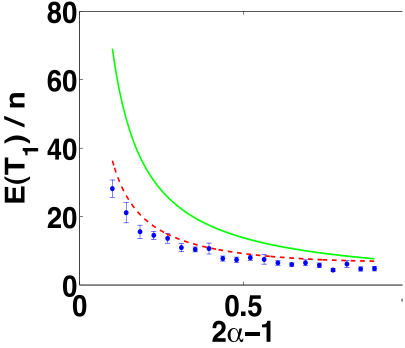

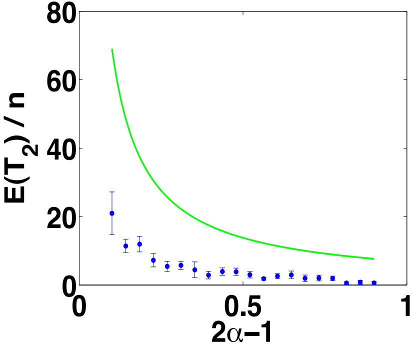

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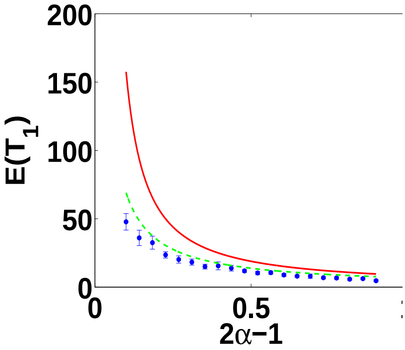

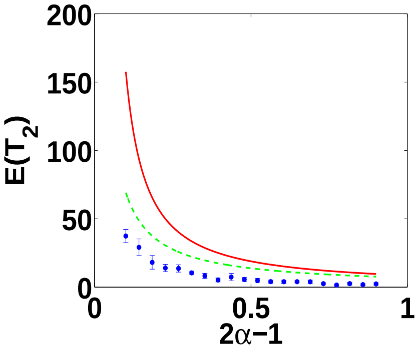

Path and Ring

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Outline

Convex Optimization [Boyd, Diaconis, Xiao '04]

For technical reasons, let us assume

$$Q = P - I_n$$

where P is a symmetric stochastic matrix.

Eigenvalue (convex) optimization

Minimize the time it takes majority consensus to converge, i.e.

$$\begin{aligned} \text{minimize} \quad & \lambda_2(P) = \sup\{x^T P x \mid x^T x = 1, x^T 1 = 0\} \\ \text{subject to } P_{ij} \geq 0, \quad & P_{ij} = 0 \text{ if } i, j \notin E \\ \text{and} \quad & \sum_j P_{ij} = 1, \forall i \end{aligned}$$

Subgradient method

- Let u be the eigenvector associated with $\lambda_2(P)$.
- Let E^ℓ, $\ell = (i, j)$ an edge in the graph such that

$$E^\ell_{ij} = E^\ell_{ji} = 1, \; E^\ell_{ii} = E^\ell_{jj} = -1$$

- The subgradient of the objective function $\lambda_2(P)$ is

$$g(P) = \left(u^T E^1 u, \ldots, u^T E^m u\right)$$

- In particular, for $\ell = (i, j)$

$$u^T E^\ell u = (u_i - u_j)^2$$

- To compute eingenvector we could use Lanczos method or recent distributed algorithms [Kempe-McSherry ’08].

Projected subgradient method [Bertsekas '99]

$k \leftarrow 1$

repeat

```
Subgradient Step
```

Calculate $g^{(k)}$ and update $P \leftarrow P - \beta_k g^{(k)}$, β_k step size, $\beta \rightarrow 0$, $\sum_k \beta_k \rightarrow \infty$

```
Sequential Projection
   Projection onto non-negative orthant
```

$P_\ell \leftarrow \max\{P_\ell, 0\}$, $\ell = 1, ..., m$

For each node $i = 1, ..., n$, $\mathcal{L}(i) = \{\ell |$ edge ℓ connected to $i\}$

```
   Projection onto half-spaces
```

While $\sum_{\ell \in \mathcal{L}(i)} P_\ell > 1$

$\mathcal{L}(i) \leftarrow \{\ell | \ell \in \mathcal{L}(i), P_\ell > 0\}$

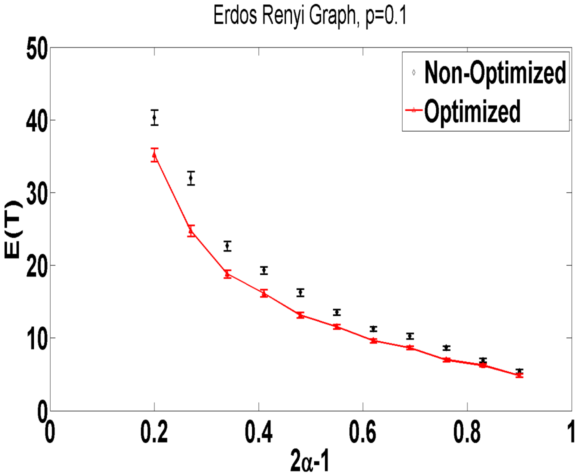
$\gamma \leftarrow \min\left\{\min_{\ell \in \mathcal{L}(i)} P_\ell, \left(\sum_{\ell \in \mathcal{L}(i)} P_\ell - 1\right) / |\mathcal{L}(i)|\right\}$

$P_\ell \leftarrow P_\ell - \gamma, \ell \in \mathcal{L}(i)$

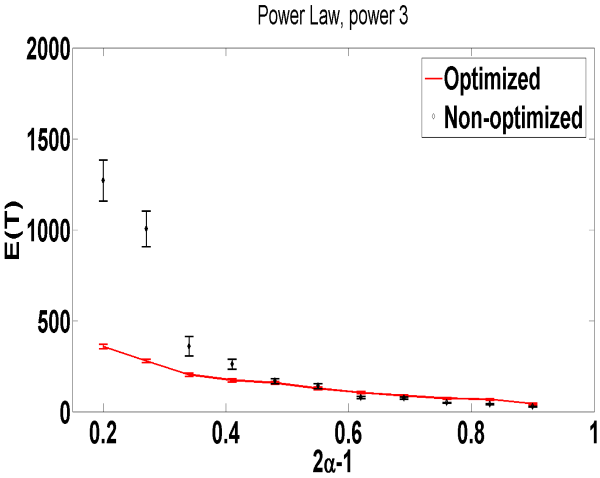
$k \leftarrow k + 1$

It can be implemented in a distributed fashion [Boyd et al '06].

ER-graph [Babaee, Draief'13+]



Preferential attachment [Babaee, Draief'13+]



Summary

- Algorithms for solving Majority consensus
- Performance: memory, error, time to convergence
- Time to convergence related to spectral properties of rate matrix
- Speedingup convergence via convex optimisation

Future Work

- Lower-bounds of convergence time
  - O. Ayaso, D. Shah and M. Dahleh, Information Theoretic Bounds for Distributed Computation over Networks of Point-to-Point Channels, IEEE IT, 2010.
  - M. Abdullah, M. Draief, Consensus on the Initial Global Majority by Local Majority Polling for a Class of Sparse Graphs, Arxiv1209.5025, 2013.
- Trade-off between memory, error, time to convergence.
- Distributed spectral computations
  - David Kempe, Frank McSherry: A Decentralized Algorithm for Spectral Analaysis, Journal of Computer and System Sciences, 2008.
  - S. Korada, A. Montanari, and S. Oh, Gossip PCA, Sigmetrics 2011.