Additional question on conformal maps

(Please, only do it if you like it!)

The aim of this (additional) excercise is to show the following:

> *Let S and $\widetilde{S}$ be two surfaces. Assume that the map $f : S \to \widetilde{S}$ preserves the angles between the curves. Then f is conformal.*

It would be the best if you could find the proof to that yourself, just using the definitions we introduced.
If you do not know how to start that you may follow the steps below.
Please, tell me if you know a shorter solution!

(a) Let $\{\boldsymbol{w}, \boldsymbol{w}_2\}$ be a basis of T_pS, and assume $||\boldsymbol{w}_1||_p = ||\boldsymbol{w}_2||_p = 1$, $\langle \boldsymbol{w}, \boldsymbol{w}_2 \rangle_p = 0$.
Let $\boldsymbol{f}_1 = d_pf(\boldsymbol{w}_1)$ and $\boldsymbol{f}_2 = d_pf(\boldsymbol{w}_2)$. Show that

$$\langle \boldsymbol{f}_1, \boldsymbol{f}_2 \rangle_{f(p)} = 0.$$

(b) Consider any two vectors $a\boldsymbol{w}_1 + b\boldsymbol{w}_2$ and $c\boldsymbol{w}_1 + d\boldsymbol{w}_2$ in T_pS. Write, what it means that the angle between these vectors is preserved by d_pf.

(c) Denote $q = \frac{\langle \boldsymbol{f}_2, \boldsymbol{f}_2 \rangle_{f(p)}}{\langle \boldsymbol{f}_1, \boldsymbol{f}_1 \rangle_{f(p)}}$. Show that

$$\frac{(ac+bd)^2}{(a^2+b^2)(c^2+d^2)} = \frac{(ac+bdq)^2}{(a+b^2q)(c^2+d^2q)} \tag{1}$$

for any $a, b, c, d \in \mathbb{R}$ such that $(a, b) \neq (0, 0)$ and $(c, d) \neq (0, 0)$.

(d) Assuming that $(a, b) = (1, 0)$, derive from the formula (1) that $q = 1$.

(e) Assuming that $q = 1$, show that f is conformal.
Find $\lambda(p)$ (in terms of the objects introduced before).