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Differential Geometry 22.04.2020

Questions for Revision Lecture

1. Let Surfaces S_1 and S_2 be given by

$$S_1 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = z^2\} \qquad S_2 = \{(x, y, z) \in \mathbb{R}^3 \mid x + 2z = 1\}.$$

   Let $\boldsymbol{\alpha} = S_1 \cap S_2$ be the set obtained by the intersection of the surfaces.

   (a) Parametrise $\boldsymbol{\alpha}$ so that $\boldsymbol{\alpha}$ is a regular curve. Compute its curvature and torsion.

   (b) Find the evolute of $\boldsymbol{\alpha}$.

   (c) Find the vertices of $\boldsymbol{\alpha}$. Does $\boldsymbol{\alpha}$ have inflection points?

2. Let $\boldsymbol{\alpha} : I \to R^3$ be a curve parametrised by arc length with curvature $\kappa(s) \neq 0,\ s \in I$. Let Π be a plane satifying both of the following conditions:

   (i) Π contains the tangent line at s.

   (ii) Given any neighborhood $J \subset I$ of s, there exist points of $\boldsymbol{\alpha}(J)$ in both sides of Π.

   Prove that Π is the osculating plane of $\boldsymbol{\alpha}$ at s.
   Hint: use the local canonical form of $\boldsymbol{\alpha}$.

3. Let $S(u, v) = (v \cos u, v \in u, \cosh v)$. Let R be the part of $S(u, v) = (x(u, v), y(u, v), z(u, v))$ given by $x^2 + y^2 \leq 2$ and $x \geq 0$. Verify the Gauss-Bonnet theorem for region R on S.

4. Let $C \in \mathbb{R}^3$ be a cylinder given by $x^2 + y^2 = 1$. Is there a self-intersecting geodesic on C?