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Feedback 1-2

General comments:

- Please, make sure you write your name c l e a r l y.
- Check your answers against common sense: length of a curve should not be negative, for non-trivial curves it should not be zero!
- Check what your answer means geometrically. In particular, if you study a plane curve and get $\kappa = const$, is your curve really a circle or a line?

Question 1.3: When computing the length, check the answer:

- Length of a curve can not be negative! If it is, probably, you are not integrating a positive function (the norm $||\cdot||$ of the tangent vector), or you have wrong limits.
- Length of non-trivial curve can not zero. In many solutions it was zero by the following reason: when computing the integral, students changed the variable u to $v = 1 - \cos u$, but this function is not monotone on the interval $(0, 2\pi)$. In this case, it would make sense to decompose the interval into several subintervals where the function is monotone. Moreover, in this question the curve is symmetric with respect to the line $x = 0$, so it suffices to compute the integral on the interval $(0, \pi)$ and double the answer.
- Another popular answer was $8\pi r^2$ (or similar). This one looks harmless (at least positive), however, it is still obviously wrong: the length can not be proportional to r^2! This answer usually means that instead of integrating the norm of the vector one integrates the square of it.

Question 1.4:

- Most solutions of part (a) were correct.
  Concerning part (b), many works used the following wrong reasoning:

  "u_0 is a point, hence $\boldsymbol{\alpha}(u_0)$ is a constant, so $\frac{d}{du}||\boldsymbol{\alpha}(u_0)|| = 0$."

  However, it does not make sense to differentiate the value at u_0 - it is not a function of u! Instead of that one should consider $\frac{d}{du}||\boldsymbol{\alpha}(u)||$ (which is a function) and take its value at u_0.
- The reasoning above is also *obviously wrong* as it does not use that the point u_0 is the closest point to the origin. In contrast to that $\frac{d}{du}||\boldsymbol{\alpha}(u)|| \,\big|_{u_0} = 0$ since u_0 is the point where $||\boldsymbol{\alpha}(u)||$ is minimal.
- In many solutions one considered the minimum of $||\boldsymbol{\alpha}(u)||$ (which is some square root), one can simplify this a little bit by stating that the minimum of $||\boldsymbol{\alpha}(u)||$ is attained at the same point where the minimum of $||\boldsymbol{\alpha}(u)||^2$ is.

Question 2.3:

- To simplify the computations, I would advise not to use *sec*, *cosec*, but write them using sin and cos.
- A popular mistake just one step before the answer: $(x^2)^{3/2} \neq x^3$! Indeed, the left hand side is always non-negative, in contrast to the right hand side. So, $(x^2)^{3/2} = |x^3|$.
- Some solutions came to the wrong answer $\kappa(u) = const$. Whenever you get this ask youself whether it is possible. In particular, if you have a curve on the plane, is your curve really a circle (for $const \neq 0$) or a line (for $const = 0$)?
- If you want to use the formula $\kappa = \boldsymbol{n} \cdot \boldsymbol{t}'$, check first whether your parameter is arc length!
- If you use $\kappa = \frac{\boldsymbol{\alpha}' \times \boldsymbol{\alpha}''}{||\boldsymbol{\alpha}'||^3}$ for a plane curve, you need to explain what you mean by the cross product (defined only in $\mathbb{R}^3$!) and to find the sign of κ separately.

Question 2.5(b):

- The main difficulty with this question was to notice the star.
- One popular mistake was to integrate $|g(x)'|$ instead of $||\boldsymbol{\alpha}'(u)||$, where $\boldsymbol{\alpha}(x) = (x, g(x))$.