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Feedback 15-16

Question 15.3: Almost everybody was fine with this question (however, there were some mistakes with computing Christoffel symbols).

Question 16.6(a): This question turned out to be more difficult.

- The easiest solution to the question is computation free: as the surface is doubly ruled, there are already two lines through every point of S (and, as we know, each line on any surface IS an asymptotic curve) - so, there are already two asymptotic curves through each point - and by a general consideration of the second quadratic form there can not be more than two. (see Solutions for the details).

- If you have decided for a computational solution, then you will need to make a choice of parametrisation. The complexity of the computation depend seriously on this choice (though, should remain doable in principle).

  For the doubly ruled surface - and when you want to get lines as the answer - the best choice would be to parametrise as a ruled surface.

- Most solutions contained parametrisations as a surface of revolution (which is also beneficial, since we can quickly write the differential equation in this case - in terms of derivatives of f and g).

  In this case you need:

  - to write the equation,
  - then to separate the variables (all u to the left, all v to the right),
  - then to integrate,
  - then to substitute the expressions for $u(s)$ and $v(s)$ into the parametrisation of the surface $x(f(v(s))\cos u(s), f(v(s))\sin u(s), g(v(s)))$ to the the curve.

- Some of the steps above may be not completely trivial (depending on the parametrisation you took). In particular, for some parametrisations, the integration included taking

$$\int \frac{dv}{\cosh v} = \int \frac{\cosh v dv}{\cosh^2 v} = \int \frac{d \sinh v}{\sinh^2 v + 1} = ...,$$

  while computation of the curve on the surface included using formulae like $\cos(\arctan x) = \frac{1}{\sqrt{x^2+1}}$ and $\cos(2\arctan x) = \frac{1-x^2}{1+x^2}$ (see below for the computation of the last formula).

- To show $\cos(2\arctan x) = \frac{1-x^2}{1+x^2}$, notice that

$$\tan z = \frac{\sin z}{\cos z} = \frac{\sin z}{\sqrt{1 - \sin^2 z}},$$

  from which we can find $\sin^2 z = \frac{\tan^2 z}{1+\tan^2 z}$ and $\cos(2\arctan x) = 1 - 2sin^2(\arctan x) = 1 - 2\frac{x^2}{1+x^2} = \frac{1-x^2}{1+x^2}$.