Durham University
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Feedback 5-6

General comments:

- Please, show your work, write all computations down: when I see the expression for $\boldsymbol{\alpha}'$ and then the answer for κ and τ in the next line, I don't know how to mark it (have you copied it from somewhere?). Moreover, if the ansewer is wrong, I have no chance to find the reason.

Question 5.1:

- Computing the angle via dot product $\cos\theta = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{||\boldsymbol{a}||||\boldsymbol{b}||}$, do not forget to divide by the norm of both vectors!
- Part (a): when showing that $\boldsymbol{t}'$ makes the same angle with $(0,0,1)$, you do not need to justify the choice of the vector $(0,0,1)$, it is enough to guess (but of course, you need to show that this vector works).
- In part (b), "If and only if" statement should have the proof for both sides. Even if one of the sides is almost trivial, you need to state this (otherwise, I think you forgot about the other side).
- Sometimes (as in this question), both sides of "A holds if and only if B" statement may be proved simultaneously, by providing a sequence of equivalent statements connecting A to B. In this case, please, write explicitely that you are having equivalences, and hence, proved both ways.

Question 5.3:

- Most solutions computed κ and τ separately, by by direct formulae. However, the computation for κ is short and τ requires more work. Instead of doing this work, one could use that $\boldsymbol{\alpha}$ is a generalised helix, and so κ/τ is a constant.
- "$g(u)$ is proportional to $f(u)$" means that $g(u) = const \cdot f(u)$.
(If to mean $g(u) = c(u)\cdot f(u)$ instead, then anything is proportional to anything else).
- In some solutions, computations used Serret-Frenet formulae and arrived to wrong answer: proportionality to e^{5u}. This is because it is impossible to use Serret-Frenet in this question, since the curve is not parametrised by arc length.

Question 6.5:

- This question is a straightforeward verification of the definition. The only difficulty here was to carefully do all the parts (not forgetting to justify all statements).
- One of the students asked at the end of his solution:

  Question: *how do we prove that $\boldsymbol{x}$ is a homeomorphism, if we can not easily find $\boldsymbol{x}^{-1}$?*

  Answer: we do not know how to do it then in general.
- The aim of this question was to see that the detailed verification of the defintion of local parametrisation is a lot of work even in the simplest case. That is why we need statements like Proposition 6.2 (about graphs of functions) or Proposition 6.7 (about level sets).