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Feedback 7-8

Question 7.1:

- There can be different parametrisations depending on the choice of the curve $\boldsymbol{\alpha}(v)$. Some of them lead to easier solutions and some for longer computations.

Question 7.4:

- Check the definitions before starting to compute! It seems, almost nobody in the course remembers what is a surface ...

  Definition 6.1. A subset $S \subset \mathbb{R}^3$ is a *regular surface* if for every point $p \in S$ there exists an open set $V \in \mathbb{R}^3$ containing p and a local parametrisation $x : U \to S \cap V$ where U is an open subset of $\mathbb{R}^2$.

  (OK, formally speaking the question should have said "regular surface", but we have not used the word "surface" in any other way in the course).

- Checking regular values is not a criterion, but just one of the methods to prove that something is a regular surface. When it fails, it does not mean you are not dealing with a regular surface. It means you should probably think about other methods.

- So, the function $(x, y, z) = (x + y + z - 1)^2$ gives an example of that: 0 is not a regular value, however the equation $f(p) = 0$ defines a plane, which is clearly a regular surface (since it is a graph of a function, or since one can easily construct a local parametrisation).

- Every value $c < 0$ is a regular value, so the preimage of it is a regular surface (even if it is an empty set!). Indeed, for every point p in this empty set we can find what we need.

- The aim of part (c) was to think about geometry of the level sets, so, I have expected to see a geometric answer (which is very simple in this case).

- In part (d) (in contrast to part (b)), the singular level set $f(p) = 0$ is not a surface by topological reasons (near the point $(0, 0, 0)$ the level set does not look as a disc). However, we have no tools to really prove that it is not. (Still, we can at least consider this case, think about its geometry/topology and give expected answer).

Question 8.2:

- This question has many easy solutations (and it is almost impossible to get it wrong).