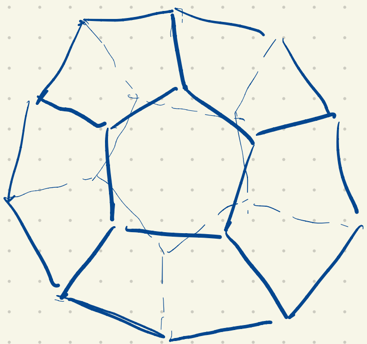
How to construct the vertices of regular icosohedron / dodecahedron on S^2 with spherical ruler & compass?



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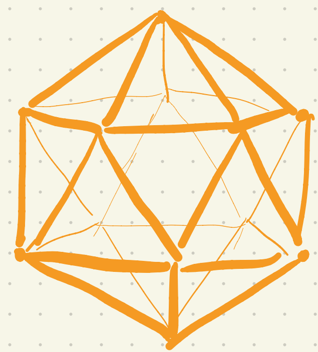
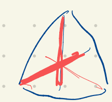


?

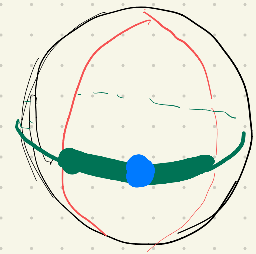
1. We can construct regular tetrahedron:

(i.e. construct triangle with $(\alpha,\beta,\gamma) = \left(\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}\right)$)

- Draw any regular triangle



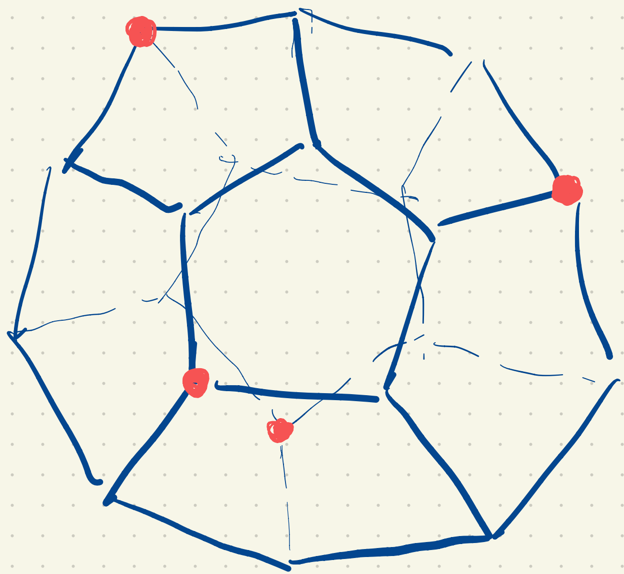
- Construct angle $\frac{2\pi}{3}$
- Construct length $\frac{2\pi}{3}$;
- Construct length $\pi/3$



- Construct triangle with $(a,b,c) = \left(\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3}\right)$
- Take polar to get triangle with $(\alpha,\beta,\gamma) = \left(\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}\right)$

1. We can construct regular tetrahedron, (see Problems Class 3).

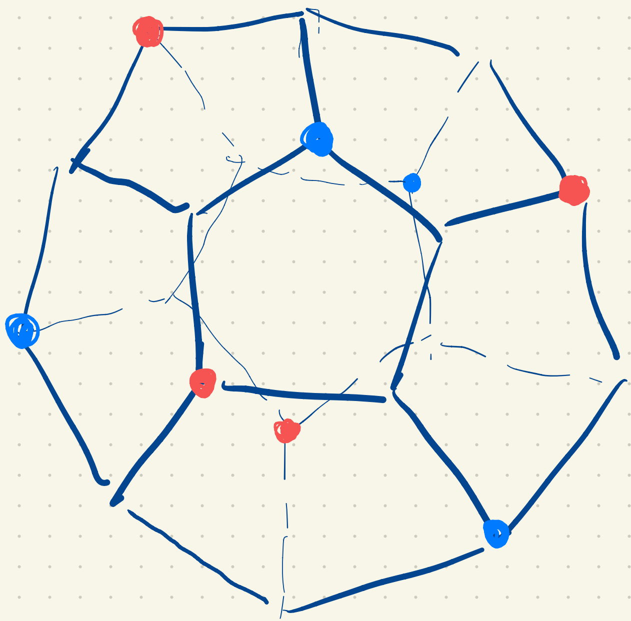
2. Embedd it into the dodecahedron:



(which we don't know how to construct yet!)

1. We can construct regular tetrahedron, (see Problems Class 3).

2. Embedd it into the dodecahedron:

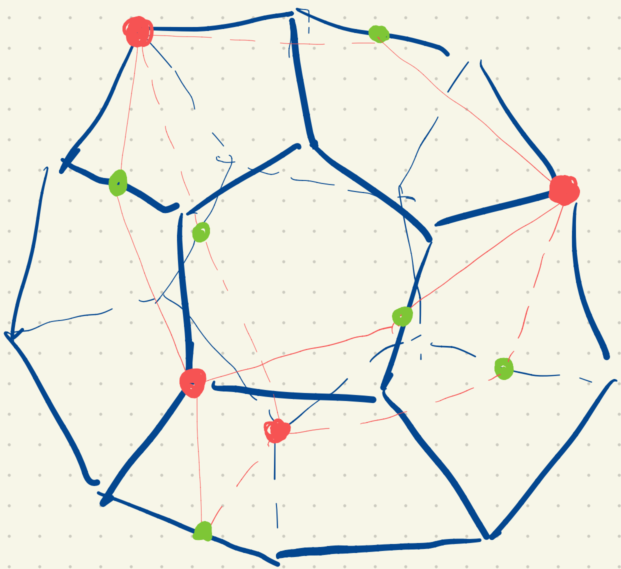


3. Draw the vertices of the dual tetrahedron.

(as centres of faces for the first one; they are also antipodal to the vertices of initial tetrahedron)

1. We can construct regular tetrahedron.
(see Problems Class 3).

2. Embed it into the dodecahedron:



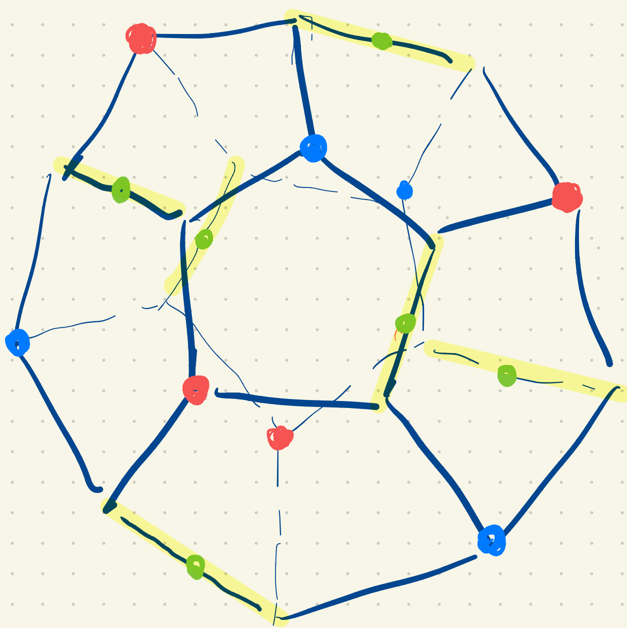
3. Dual tetrahedron.

4. Given vertices of the tetrahedron, can construct midpoints of six edges (as midpoints of edges of the tetrahedron) get an octahedron.

1. We can construct regular tetrahedron.

(see Problems Class 3).

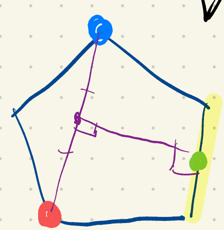
2. Embed it into the dodecahedron:



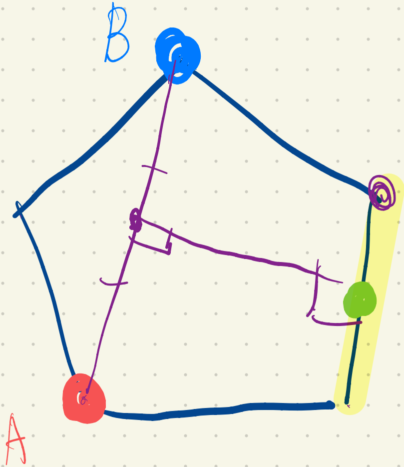
3. Dual tetrahedron.

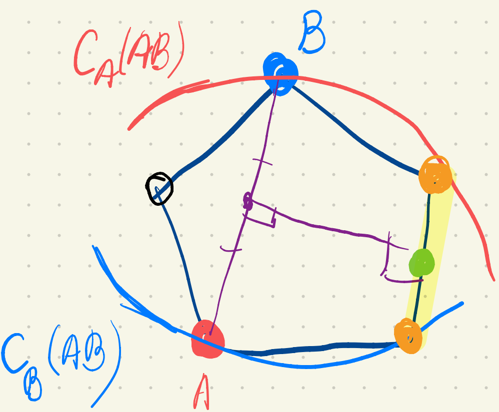
4. midpoints of six edges.

5. construct lines containing vertices of the octahedron



(from a pentagon where we know 3 pts)





6. In a face, consider a circle of radius AB:

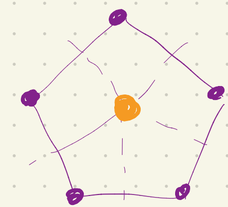
$C_B(AB)$ and $C_A(AB)$:

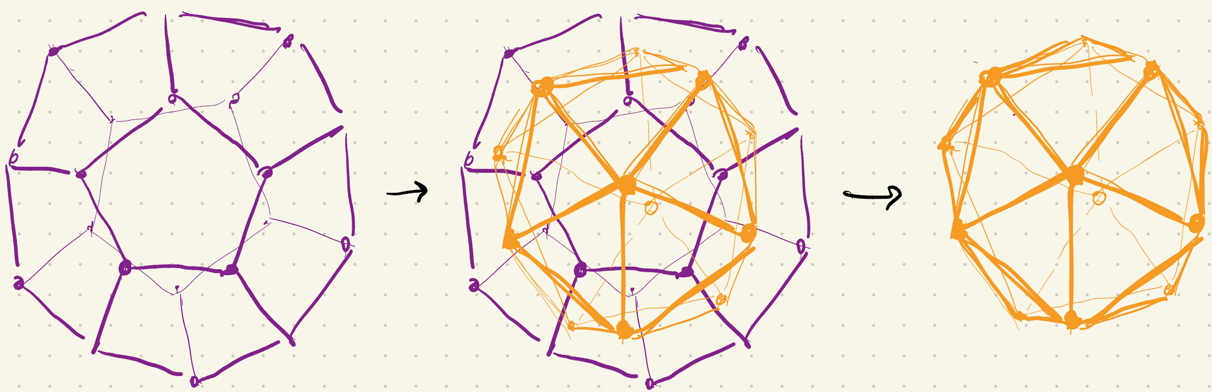
in intersection with the opposite side get a vertex

- The last vertex is on distance AB from both new vertices

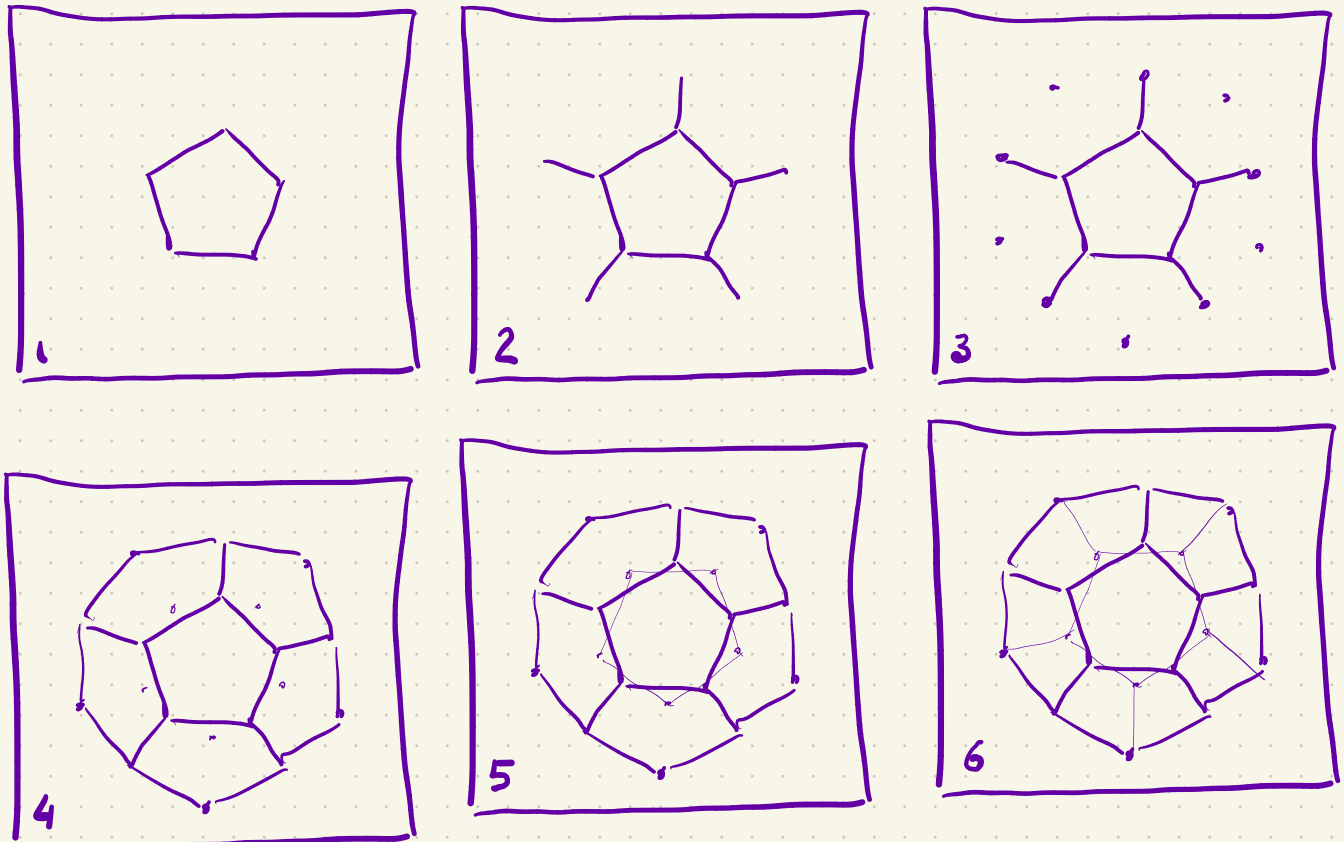
- This way we reconstruct vertices of all 12 faces!

- We constructed vertices of dodecahedron.
- Vertices of icosahedron can be constructed as centres of pentagonal faces:





Bonus 1 How to draw a dodecahedron:



Bonus 2: How to draw an icosahedron: