Geometry III/IV

Exercises: Week 11, Jan 2013

Part A

Problem 1. A *circle* $C_{A,r}$ of radius r centered at A is a set of points on distance r from A.

Show that any spherical circle on a sphere $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = 1\}$ is represented by a Euclidean circle.

Problem 2. Given ASA congruence law for spherical triangles, derive the SAS law. Hint: use the polar triangle.

Problem 3.

(a) Find the area of a spherical triangle with angles $\frac{\pi}{2}, \frac{\pi}{3}$ and $\frac{\pi}{3}$. Which part of the area of the whole sphere does it make?

(b) The same question for the triangle with angles $\frac{\pi}{2}, \frac{\pi}{3}$ and $\frac{\pi}{4}$.

Problem 4. (a) Find the area of a spherical quadrilateral with angles $\alpha, \beta, \gamma, \delta$.

(b) Given angles of a spherical n-gone, find its area.

Problem 5. For a spherical triangle with angles $\frac{\pi}{2}, \frac{\pi}{4}, \frac{2\pi}{3}$ on the unit sphere find the length of the side opposite to the angle $\frac{2\pi}{3}$.

Part B

Problem 6. Let X be an infinitely long cylinder $X = (\varphi, z)/_\sim$ where $(\varphi, z) \in \mathbb{R}^2$ and $\varphi_1 \sim \varphi_2$ if and only if $\varphi_1 - \varphi_2 \in 2\pi k$, $k \in \mathbb{Z}$. Given the Euclidean metric on $\mathbb{R}^2$, this factorisation defines a Euclidean metric on the cylinder X (in other words, any small piece of X is Euclidean).

(a) Find a closed geodesic on X;

(b) Find an open geodesic on X.

Problem 7. A *self-polar* triangle is a triangle polar to itself.

(a) Show that a self-polar triangle does exist.

(b) Show that all self-polar triangles are congruent.
(Hint: use the definition of polar triangles).

Part C

Problem 8. (for students that are enrolled in MATH4141, Geometry IV.)
Compare the notion of polarity for sphere with the notion of polarity you know from extra reading material.