Geometry III/IV

Exercises: Week 12, Jan 2013

Part A

Problem 1. For a spherical triangle with angles $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}$ on the unit sphere find the length of the side opposite to the angle $\frac{\pi}{3}$.

Problem 2. Prove the formulas for a spherical triangle with right angle γ:

(a) $\tan a = \tan\alpha \sin b$ (b) $\tan a = \tan c \cos\beta$.

Hint: use both sine and cosine laws.

Problem 3. Let T be a spherical triangle with three right angles. Let r and R be the radii of the inscribed and superscribed circles for T. Find the ratio $\sin R/\sin r$.

Problem 4. Find a Möbius transformation which maps $1, 1+i, \infty$ to $0, \infty, 1$, respectively.

Problem 5. Find an inverse for the Möbius transformation $f(z) = \frac{z+1}{iz}$.

Part B

Problem 6. Let ABC be a triangle with angles $\angle A = \frac{\pi}{2}$, $\angle B = \frac{\pi}{3}$, $\angle C = \frac{\pi}{5}$. Let ϕ be a clockwise rotation by $\frac{2\pi}{3}$ around the point B and let ψ be a clockwise rotation by $\frac{2\pi}{5}$ around the point C. What can you say about the isometry $\phi \circ \psi$?

Hint: write ϕ and ψ as compositions of two reflections.

Problem 7.
(a) Recall the classification of Euclidean isometries (4 types). Can you write each Euclidean isometry as a composition of reflections?
(b) Find the minimal number which turns the following into the correct statement:

"Any Euclidean isometry may be written as a composition of at most .... reflections".

Problem 8.
(a) Find an example of orientation-reversing isometry of the sphere which is not a reflection.
(b) Can you find a one-parametric family of isometries of this type? Hint: recall that an isometry of this type is a composition of three reflections.
(c)* Classify orientation-reversing isometries of the sphere.