Geometry III/IV

Exercises: Week 13, Feb 2013

Part A

Problem 1. What type is the transformation $1/z$?
(Hint: parabolic or non-parabolic? if non-parabolic, then elliptic, hyperbolic or loxodromic?)

Problem 2. Write the following transformations as compositions of inversions and reflections:
  (a) az, $a \in \mathbb{R}$;   (b) $z + b$, $b \in \mathbb{C}$;   (c) $1/z$.

Problem 3. Let I be an inversion with respect to the unit circle $|z| = 1$. Find the image $I(l)$ of the line l given by the equation $x = 2$.

Problem 4. Find the cross-ratio $[1, i, -1, -i]$.

Problem 5. Do the points $-1 - 2i$, $-1 + 2i$, $3 + i$, $3 - i$ lie on one line or circle?

Problem 6. Show that a finite order Möbius transformation is elliptic.
(g is of finite order if $g^n = Id$ for some positive integer n).

Part B

Problem 7. Find a parabolic Möbius transformation preserving the point $z = 1$.