Geometry III/IV

Exercises: Week 16, Feb 2013

Part A

Problem 1. Let ABC be a triangle. Let $B_1 \in AB$ and $C_1 \in AC$ be two points such that $AB_1C_1 = \angle ABC$. Show that $\angle AC_1B_1 > \angle ACB$.

Problem 2. Show that there is no "rectangle" in hyperbolic geometry (i.e. no quadrilateral has four right angles).

Part B

Problem 3. Given α, β, γ such that $\alpha+\beta+\gamma < \pi$ show that there exists a hyperbolic triangle with angles α, β, γ.

Problem 4. Let l_1 and l_2 be two hyperbolic lines. Show that exactly one of the following three possibilities hold:

- either l_1 intersect l_2,
- or l_1 is parallel to l_2,
- or there exists a unique line l orthogonal to both of l_1 and l_2.

Problem 5. Show that there exists a hyperbolic pentagon with five right angles.

Problem 6. An *ideal* triangle is a hyperbolic triangle with all three vertices an the absolute.

(a) Show that all ideal triangles are congruent.

(b) Show that three altitudes of an ideal triangle intersect in one point. (An *altitude* is a straight line passing through a vertex and orthogonal to the opposite side).

(c) Let O be a point of intersection of altitudes of an ideal triangle XYZ. Let P be a foot of the altitude XP ($P \in YZ$, $XP \perp YZ$). Find the angles of the triangle YOP. Find the area of YOP. Find $d(O, P)$.

(d) Show that an ideal triangle has an inscribed circle. Find its radius.

(e) Let c be a circle inscribed in a triangle ABC. Show that the radius of c does not exceed $\frac{2}{\sqrt{3}}$.