Geometry III/IV

Exercises: Week 17, Feb 2013.

Part A

Problem 1. In the Klein disk model draw two parallel lines, two ultraparallel lines, an ideal triangle, a triangle with angles $(0, \frac{\pi}{2}, \frac{\pi}{3})$.

Problem 2. Consider the two-sheet hyperboloid model
$\{u = (u_1, u_2, u_3) \in R^{2,1} \mid (u, u) = -1 \ u_3 > 0\}$, where $(u, u) = u_1^2 + u_2^2 - u_3^2$.

(a) For the vectors

$$\begin{array}{lll} v_1 = (2, 1, 2) & v_2 = (0, 1, 2) & v_3 = (3, 4, 5) \\ v_4 = (1, 0, 0) & v_5 = (0, 1, 0) & v_6 = (1, 1, 2) \end{array}$$

decide if v_i corresponds to a point in $\mathbb{H}^2$, or a point in the absolute, or a line in $\mathbb{H}^2$. (Hint: you will find two points, one point of ∂H^2 and three lines).

(b) Find the distance between the two points of $\mathbb{H}^2$ described in (a).

(c) Which pair the lines in (a) is intersecting? Which lines are parallel? Which are ultraparallel?

(d) Find the distance between the pair of ultraparallel lines in (a).

(e) Does any of the points in (a) lie on any of the three lines?

(f) Find the angle between the pair of intersecting lines.

Part B

Problem 3. In the Klein model draw a pentagon with five right angles.

Problem 4. Show that three altitudes of a hyperbolic triangle either have a common point or are pairwise parallel or there is a unique line orthogonal to all three altitudes. (Hint: place a triangle ABC in the Klein model so, that the altitudes of ABC were represented by (Euclidean) altitudes of the corresponding Euclidean triangle, then use the following statement: *"Three altitudes of Euclidean triangle have a common point"*).