# Geometry III/IV

Exercises: Week 18, March 2013

This is a marked assignment! Due to Friday, March 15.

**Problem 1.** Let $a$ and $b$ be two vectors in the hyperboloid model such that $(a, a) > 0$ and $(b, b) > 0$. Let $l_a$ and $l_b$ be the lines determined by equations $(x, a) = 0$ and $(x, b) = 0$ respectively. And let $r_a$ and $r_b$ be reflections with respect to $l_a$ and $l_b$.

(a) For $a = (0, 1, 0)$ and $b = (1, 0, 0)$ write down $r_a$ and $r_b$.
Find $r_b \circ r_a(v)$, where $v = (0, 1, 2)$.

(b) What type is the isometry $\phi = r_b \circ r_a$ for $a = (1, 1, 1)$ and $b = (1, 1, -1)$? (Hint: you don't need to compute $r_a$ and $r_b$).

(c) Find an example of $a$ and $b$ such that $\phi = r_b \circ r_a$ is a rotation by $\pi/2$.

**Problem 2.** Draw two horocycles $h_1$ and $h_2$ centred at the same point and such that $d(h_1, h_2) = 1$ (where $d(p, q) = \min\limits_{P \in p, Q \in q} d(P, Q)$).

**Problem 3.** Let $XYZ$ be an ideal triangle (i.e. a triangle with $XYZ \in \partial \mathbb{H}^2$). Let $H_x, H_y$ and $H_z$ be the foots of its altitudes. Find $d(H_x, H_y)$.