Geometry III/IV

Exercises: Week 19, March 2013

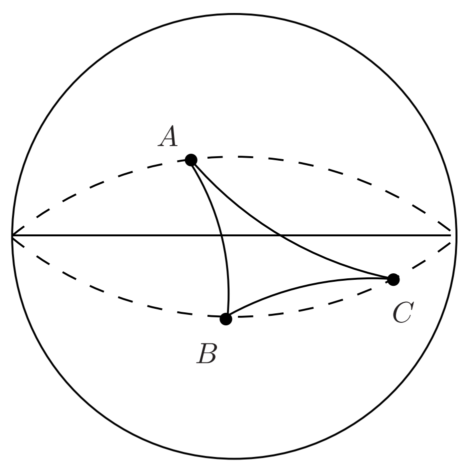
Warning: this set of exercises is a bit more difficult than one you will find in the Exam!

Problem 1. Find the circumference of a hyperbolic circle of radius R.
Hint: inscribe into the circle a regular n-gon, find its perimeter, find the limit of the perimeter as n tends to infinity.

Problem 2. Find the circuference of a spherical circle of radius R.
Hint: use the same method as in Problem 1.

Problem 3. Draw two horocycles h_1 and h_2 centred at the same point and such that $d(h_1, h_2) = 1$ (where $d(p, q) = \min\limits_{P \in p, Q \in q} d(P, Q)$).

Problem 4. Let l be a hyperbolic line and A, B and C be points on some equidistant curve to the line l (i.e. A, B, C lie on the same distance from l), so that A is separated from B and C by the line l as in the diagram below. Show that the area of the triangle ABC does not depend on the choice of A on the equidistant curve.
Hint: draw the orthogonal projections from the points A,B, C to the line l.



Problem 5. Define an equidistant curve for a line in Euclidean plane and for a spherical line. Prove the statement described in Problem 4 for the cases of Euclidean plane and a sphere.
Hint: your solution for Problem 4 will probaly work here.