Geometry III/IV

Time and place: Fr 13:00, 15:00 CG60
Course webpage: http://www.maths.dur.ac.uk/users/anna.felikson/Geometry/

Instructor: Anna Felikson
Office: CM235; Phone: 334-4158
e-mail: anna.felikson@durham.ac.uk
Office hours: Mon 12:00-13:00

Spherical geometry – outline

1. Distance and geodesics.
   - $d(A, B) = R\angle AOB$ (for a sphere of radius R centred at O).
   - Geodesics are great circles.
2. Polar correspondence.
   - Equator $\to$ union of two poles; any pole $\to$ corresponding equator.
   - If $A \in l$ then $Pol(l) \in Pol(A)$ (where A is a point and l is a line).
   - Polar triangle: $A'B'C'$ is polar for ABC if A' is polar to the line $\overline{BC}$ containing the side BC (chosen so that $\overline{BC}$ does not separate A from A') and similar properties hold for B' and C'.
   - Bipolar theorem: if $A'B'C' = Pol(ABC)$ then $Pol(A'B'C') = ABC$.
   - Angles and sidelengths of polar triangles:
     $(\alpha', \beta', \gamma') = (\pi - a, \pi - b, \pi - c), \quad (a', b', c') = (\pi - \alpha, \pi - \beta, \pi - \gamma)$.
3. Spherical triangles.
   a. Four theorems of congruence of spherical triangles: ASA, SAS, SSS, AAA.
   b. Area of a spherical triangle: $S_{ABC} = (\alpha + \beta + \gamma - \pi)R^2$
      where R is the radius of the sphere.
      In particular, $\alpha + \beta + \gamma > \pi$.
   c. Sine and cosine theorems:
      – sine theorem $\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$
      – cosine thm: $\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$
      – second cosine thm: $\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a$
4. Isometries of the sphere:
   - Any isometry of the sphere is uniquely determined by images of three points.
   - Isometries act transitively on the sphere.
   - Isometry group of the sphere is generated by reflections.
   - Any isometry is a product of at most three reflections.
   - Any orientation preserving isometry is a rotation (a product of two reflections).
   - Any orientaion reversing isometry is either a reflection or a product of three reflections.