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Formula sheet

Sine and cosine laws:

	sine law	cosine law
S^2	$\frac{\sin a}{\sin\alpha}=\frac{\sin b}{\sin\beta}=\frac{\sin c}{\sin\gamma}$	$\cos a=\cos b\cos c+\sin b\sin c\cos\alpha$ $\cos\alpha=-\cos\beta\cos\gamma+\sin\beta\sin\gamma\cos a$
$\mathbb{E}^2$	$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$	$a^2=b^2+c^2-2bc\cos\alpha$
$\mathbb{H}^2$	$\frac{\sinh a}{\sin\alpha}=\frac{\sinh b}{\sin\beta}=\frac{\sinh c}{\sin\gamma}$	$\cosh a=\cosh b\cosh c-\sinh b\sinh c\cos\alpha$

Angle of parallelism in hyperbolic geometry:
For a point on distance a from the line, the angle of parallelism φ satisfies

$$\sin\varphi=\frac{1}{\cosh a}$$

Distance formula in the upper half-plane model of hyperbolic geometry:

$$\cosh d(u,v)=1+\frac{|u-v|^2}{2Im(u)Im(v)}$$

Distance formula in the hyperboloid model of hyperbolic geometry:
For $u,v\in\mathbb{R}^{2,1}$, let $Q=|\frac{(u,v)^2}{(u,u)(v,v)}|$. Then

if $(u,u)<0$, $(v,v)<0$ then $Q=\cosh^2 d(pt,pt)$

if $(u,u)<0$, $(v,v)>0$ then $Q=\sinh^2 d(pt,line)$

if $(u,u)>0$, $(v,v)>0$ then $Q<1\Rightarrow$ intersecting lines, $\quad Q=\cos^2\alpha$;
$Q=1\Rightarrow$ parallel lines;
$Q>1\Rightarrow$ ultraparallel lines, $Q=\cosh^2 d(line,line)$