

Hints 15-16

15.1. Part (c) of the question contains a **misprint**:
the radius should not exceed $\operatorname{arcosh}(2/\sqrt{3})$ (rather than $2/\sqrt{3}$).
To solve the corrected question, compute in the upper half-plane (don't forget first to place the triangle nicely).

15.2. Square the required expression, express \tanh^2 through \cosh^2 and use the distance formula to get the latter. Also, to get \sin^2 use the formula $\sin^2 \alpha = \frac{(2k \cos \varphi)^2}{(1-k^2)^2 + 4k^2 \cos^2 \varphi}$ obtained in the Problems Class.

15.3. As was pointed out by one of you in the problems class, there is a **misprint**:
the formula should be

$$2 \sinh^2 \frac{d}{2} = \frac{|z - w|^2}{2\operatorname{Im}(z)\operatorname{Im}(w)}.$$

To solve the corrected question, use the definitions of \sinh and \cosh as half-sum of two exponents.

15.5. Take one point on the given distance from the line and apply some isometries to get more points on the same distance.

16.2. Place your triangle in the Klein model in such a way that all altitudes will be represented by the altitudes of Euclidean triangle.

16.3. To compute, place the objects so that the required distance will be a length of the segment lying in the plane $z_2 = 0$, then everything is reduced to 2-dimensional problem.

16.4. Use formulae listed in 16.3.