Dr. Anna Felikson, Durham University **Geometry**, 21.10.2014

Hints 3-4

3.1 If C and D lie on different sides with respect to AB then the segment CD intersects the line AB.

3.2 (a) Use the result of the previous question.

3.3 (*) This is a direct computation based on the definition of isometry.

3.4 This is just to apply the definition of a discrete action and of an orbit space.

3.5 (*) There are many ways to choose the group H for this question. Go for the easiest one: it will be helpful for later parts of this question.

(d) If F is a fundamental domain for G and H is a subgroup of G, then F tiles the fundamental domain for H (why?). The index [G:H] may be found as the number of the tiles.

4.1 The geodesics on X come from geodesics on $\mathbb{E}^2$ - just find the good ones.

4.2 Use lines of rational/irrational slopes on $\mathbb{E}^2$.

4.3 (*) Try to project something somewhere.

4.4 (*) This question is a bit more involved then the others. The main idea is to find a projection p of the spherical triangle to some plane, so that p will take a spherical triangle to a Euclidean one and a spherical median/altitude to a Euclidean one.

(a) Project from the centre of the sphere O to the plane ABC.

(b) Project from O to the plane Π_C tangent to the sphere at the point C. To prove that the altitudes of a spherical triangle are projected to the altitudes (of Euclidean triangle) one can use the following statement (prove the statement!):

> *Let Π_C be a plane in $\mathbb{E}^3$ tangent to the sphere at the point C.*
> *Let α be a plane, $O, C \in \alpha$ and let β be any plane through O orthogonal to α.*
> *Let $l_\alpha = \Pi_C \cap \alpha$ and $l_\beta = \Pi_C \cap \beta$ be non-empty. Then l_α is orthogonal to l_β.*

Remark 1. In fact, if ABC is a spherical triangle of a very special type then ABC has altitudes which are not concurrent! Can you find such an example? What is wrong with the proof (or more precisely, with the projection) in this case?

Remark 2. As the solution both for medians and altitudes refers to the corresponding Euclidean theorems, it looks reasonable to refresh/to learn the proofs of the Euclidean theorems. For example, one can find the proofs on the cut-the-knot.