Homework 1-2

Starred problems due on Tuesday, 21 October

1.1 In this exercise we recall basic theorems of Euclidean geometry.
For each Theorem or Corollary in the handout

(a) draw the diagram;

(b) is the statement new for you or you have already seen this statement before (say, in school)?

(c) list the statements which look new to you.
Remark: Please, prepare the list like "E6, E17, E16:cor,E20:converse",
we will fill in a short questionnaire on Tuesday, 14 October.

2.1 (*) Let $Isom^+(\mathbb{E}^2) \subset Isom(\mathbb{E}^2)$ be a group of orientation-preserving isometries of $\mathbb{E}^2$.
Show that $Isom^+(\mathbb{E}^2)$ is generated by rotations.

2.2 Show that a composition of a rotation and a translation is a rotation by the same angle. How to find the centre of the new rotation?

2.3 A *glide reflection* is a composition of a reflection with respect to a line and a translation along the same line.
Show that every composition of 3 reflections in $\mathbb{E}^2$ is a glide reflection.

2.4 (*) List all finite order elements of the group $Isom(\mathbb{E}^2)$. Justify your answer.

2.5 Let t_a be a translation by the vector a and let $R_{\alpha,z}$ be a rotation by angle α around $z \in \mathbb{C}$. What can you say about the isometry $f = R_{\alpha,z} \circ t_a \circ R_{-\alpha,z}$?

2.6 Give an example of an isometry $f : \mathbb{E}^2 \to \mathbb{E}^2$ and a set $A \subset \mathbb{E}^2$ for which

(a) $f(A) \subset A$, $f(A) \neq A$;

(b) A is a bounded set, $f(A) \subset A$, $f(A) \neq A$.

2.7 (*) Let $x = (x_1, x_2)$ be a point in $\mathbb{E}^2$ and $a = (a_1, a_2)$ be a vector. Consider the line given by the equation $(x, a) = 0$, i.e. the set of points $\{(x_1, x_2) \mid a_1x_1 + a_2x_2 = 0\}$.
Show that the transformation

$$f : x \mapsto x - 2\frac{(x,a)}{(a,a)}a$$

(a) is an isometry;

(b) preserves the line $(x, a) = 0$ pointwise;

(c) is a reflection with respect the the line $(x, a) = 0$.

(d) What is the geometric meaning of $\frac{(x,a)}{(a,a)}a$?
(It should help you to see that f is the reflection even without any computations).

2.8 (Mirror on the wall)
Assume you are 2m tall and looking at the wall mirror from 1m away. How long the mirror should be so that you could see both your toes and your head? How the answer depend on your hight? on the distance to the mirror?

References:

1. You will find (almost) all material of lectures 3 and 4 in
   - G. Jones, *Algebra and Geometry*, Lecture notes (Section 1).
2. You may also find more stuff in the spirit of lectures 3 and 4 in
   - N. Peyerimhoff, *Geometry III/IV*, Lecture notes (Section 1).
3. If you want to read more about the role of reflections in $Isom(\mathbb{E}^2)$, the perfect source is
   - O. Viro, *Defining relations for reflections. I*, arXiv:1405.1460v1.