

## Spherical, Euclidean and hyperbolic geometries

|                                   | Spherical   | Euclidean  | hyperbolic  |
|-----------------------------------|---|--|---|
| Lines                             | intersecting (2 pts of intersection)  | intersecting (1 pt)<br>parallel  | intersecting (1 pt of intersection)<br>parallel (1 pt on $\partial\mathbb{H}^2$ )<br>ultraparallel                                    |
| orientation-preserving isometries | rotation  | rotation<br>translation  | elliptic<br>parabolic<br>hyperbolic   |
| congruence of triangles: AAA      | +   | -  | +   |
| Angle sum of a triangle           | $> \pi$   | $= \pi$  | $< \pi$   |
| Area of a triangle                | $(\alpha + \beta + \gamma) - \pi$   |  | $\pi - (\alpha + \beta + \gamma)$   |
| Pythagorean thm                   | $\cos c = \cos a \cos b$  | $c^2 = a^2 + b^2$  | $\cosh c = \cosh a \cosh b$   |
| Law of sines                      | $\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$   | $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$ | $\frac{\sinh a}{\sin \alpha} = \frac{\sinh b}{\sin \beta} = \frac{\sinh c}{\sin \gamma}$  |
| Law of cosines                    | $\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$<br>$\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a$ | $a^2 = b^2 + c^2 - 2bc \cos \alpha$                                    | $\cosh a = \cosh b \cosh c - \sinh b \sinh c \cos \alpha$<br>$\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cosh a$ |
| Circumference of a circle         | $2\pi \sin R$   | $2\pi R$   | $2\pi \sinh R$  |
| Area of a disc                    | $4\pi \sin^2(\frac{r}{2})$  | $\pi R^2$  | $4\pi \sinh^2(\frac{r}{2})$   |