Formula sheet

Sine and cosine laws:

	sine law	cosine laws
S^2	$\frac{\sin a}{\sin\alpha} = \frac{\sin b}{\sin\beta} = \frac{\sin c}{\sin\gamma}$	$\cos a = \cos b\cos c + \sin b\sin c\cos\alpha$ $\cos\alpha = -\cos\beta\cos\gamma + \sin\beta\sin\gamma\cos a$
$\mathbb{E}^2$	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$	$a^2 = b^2 + c^2 - 2bc\cos\alpha$
$\mathbb{H}^2$	$\frac{\sinh a}{\sin\alpha} = \frac{\sinh b}{\sin\beta} = \frac{\sinh c}{\sin\gamma}$	$\cosh a = \cosh b\cosh c - \sinh b\sinh c\cos\alpha$ $\cos\alpha = -\cos\beta\cos\gamma + \sin\beta\sin\gamma\cosh a$

Circles:

	S^2	$\mathbb{E}^2$	$\mathbb{H}^2$
Circumference of a circle	$2\pi\sin R$	$2\pi R$	$2\pi\sinh R$
Area of a disc	$4\pi\sin^2(\frac{R}{2})$	πR^2	$4\pi\sinh^2(\frac{R}{2})$

Angle of parallelism in hyperbolic geometry:
For a point on distance a from the line, the angle of parallelism φ satisfies

$$\sin\varphi = \frac{1}{\cosh a}$$

Distance formula in the upper half-plane model of hyperbolic geometry:

$$\cosh d(u,v) = 1 + \frac{|u-v|^2}{2\mathrm{Im}(u)\mathrm{Im}(v)}$$

Distance formula in the hyperboloid model of hyperbolic geometry:
For $u, v \in \mathbb{R}^{2,1}$, let $Q = |\frac{(u,v)^2}{(u,u)(v,v)}|$. Then

if $(u,u) < 0$, $(v,v) < 0$ then $Q = \cosh^2 d(pt, pt)$

if $(u,u) < 0$, $(v,v) > 0$ then $Q = \sinh^2 d(pt, line)$

if $(u,u) > 0$, $(v,v) > 0$ then $Q < 1 \Rightarrow$ intersecting lines, $Q = \cos^2\alpha$;
$Q = 1 \Rightarrow$ parallel lines;
$Q > 1 \Rightarrow$ ultraparallel lines, $Q = \cosh^2 d(line, line)$