Hints 5-6

5.2. One can reuse the same proof as for Euclidean case
(considering perpendicular bisector and angle bisector as loci of something...).

5.3. Use polar correspondence.

5.6. Both sine and cosine laws will be useful.

6.2. Triangulate the polygon.

6.4. Use reflections.

6.5. This question is a bit more involved then the others. The main idea is to find a projection p of the spherical triangle to some plane, so that p will take a spherical triangle to a Euclidean one and a spherical median/altitude to a Euclidean one.

(a) Project from the centre of the sphere O to the plane ABC.

(b) Project from O to the plane Π_C tangent to the sphere at the point C. To prove that the altitudes of a spherical triangle are projected to the altitudes (of Euclidean triangle) one can use the following statement (prove the statement!):

Let Π_C be a plane in $\mathbb{E}^3$ tangent to the sphere at the point C.
Let α be a plane, $O, C \in \alpha$ and let β be any plane through O orthogonal to α.
Let $l_\alpha = \Pi_C \cap \alpha$ and $l_\beta = \Pi_C \cap \beta$ be non-empty. Then l_α is orthogonal to l_β.

Remark 1. In fact, if ABC is a spherical triangle of a very special type then ABC has altitudes which are not concurrent! Can you find such an example? What is wrong with the proof (or more precisely, with the projection) in this case?

Remark 2. As the solution both for medians and altitudes refers to the corresponding Euclidean theorems, it looks reasonable to refresh/to learn the proofs of the Euclidean theorems. For example, one can find the proofs on the cut-the-knot.