Dr. Anna Felikson,
Durham University

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Christmas Problems

Snowed problems to be sent to Santa Claus
by Saturday, 24 December

1. (*) In the Christmas forest.

The Christmas forest consists of trees of two colours
(see the back side).

(a) Find a group $G \subset Isom(\mathbb{E}^2)$ acting transitively on white trees and preserving the colours. Describe G by generators and relations.

(b) Santa Claus' lives in the orbit space of the G-action. How many trees are there in the orbit space?

(c) Santa asks elves to light some candles so that at least one candle could be seen from each point of the forest. How many candles do they need per a tree if

- the light travels along geodesics and
- the light can not pass through the boundary of a tree?

(d) The same question as in (c), but now the boundaries of trees are covered with ice mirrors, so that the light is reflected at the boundaries.

(e) The group G_0 of orientation preserving isometries acts on the Christmas forest taking trees to trees (but not caring about colours). If G_0 acts transitively on all trees, can you find the orbit space?

2. (*) Dual forest.

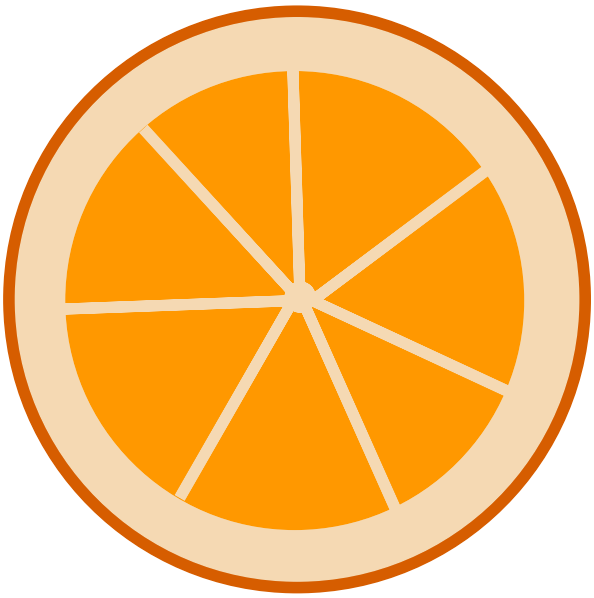
Let $\mathcal{C}$ be a circle superscribed around a Christmas tree
(see the back side again).
Sketch the domain dual to the tree with respect to $\mathcal{C}$.
How would the dual to the Christmas forest look like?

3. Buying spherical oranges.

A spherical orange is a disc on a sphere. These oranges have quite thick peel: an orange of radius r has only a $\frac{4}{5}r$-disc of pulp and the rest is peel. Oranges are good and tasty, the peel is rubbish.

a. A shop sells big and small oranges (both are tasty) for the same price per unit of total area. What costs less per unit of area of pulp, the big ones or the small ones?

b. The shop also sells Euclidean oranges for the same price. Which would you prefer to buy, spherical or Euclidean ones?



4. Cutting a triangular cake.

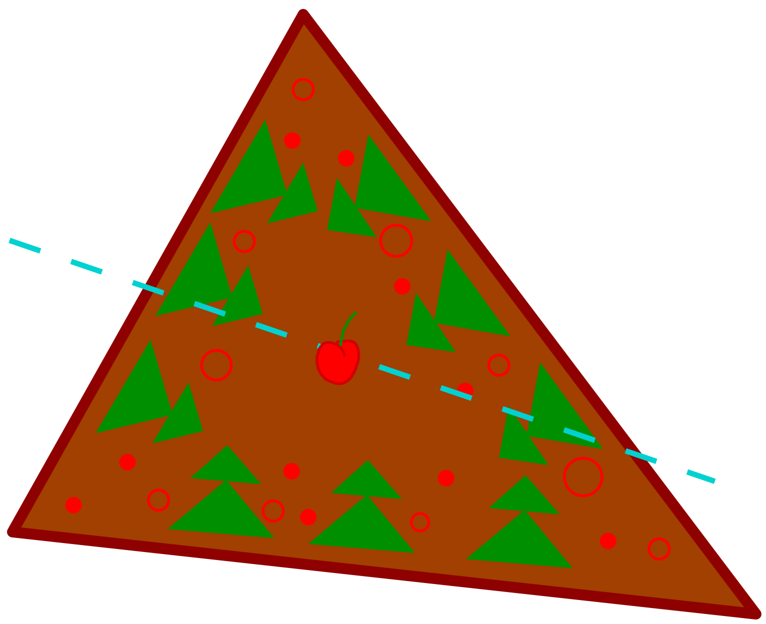
Alice and Bob have a triangular cake to share. Alice chooses a point A inside the cake, then Bob chooses a straight line through A to cut the cake in two parts. Alice gets the **smaller** part and Bob the bigger.

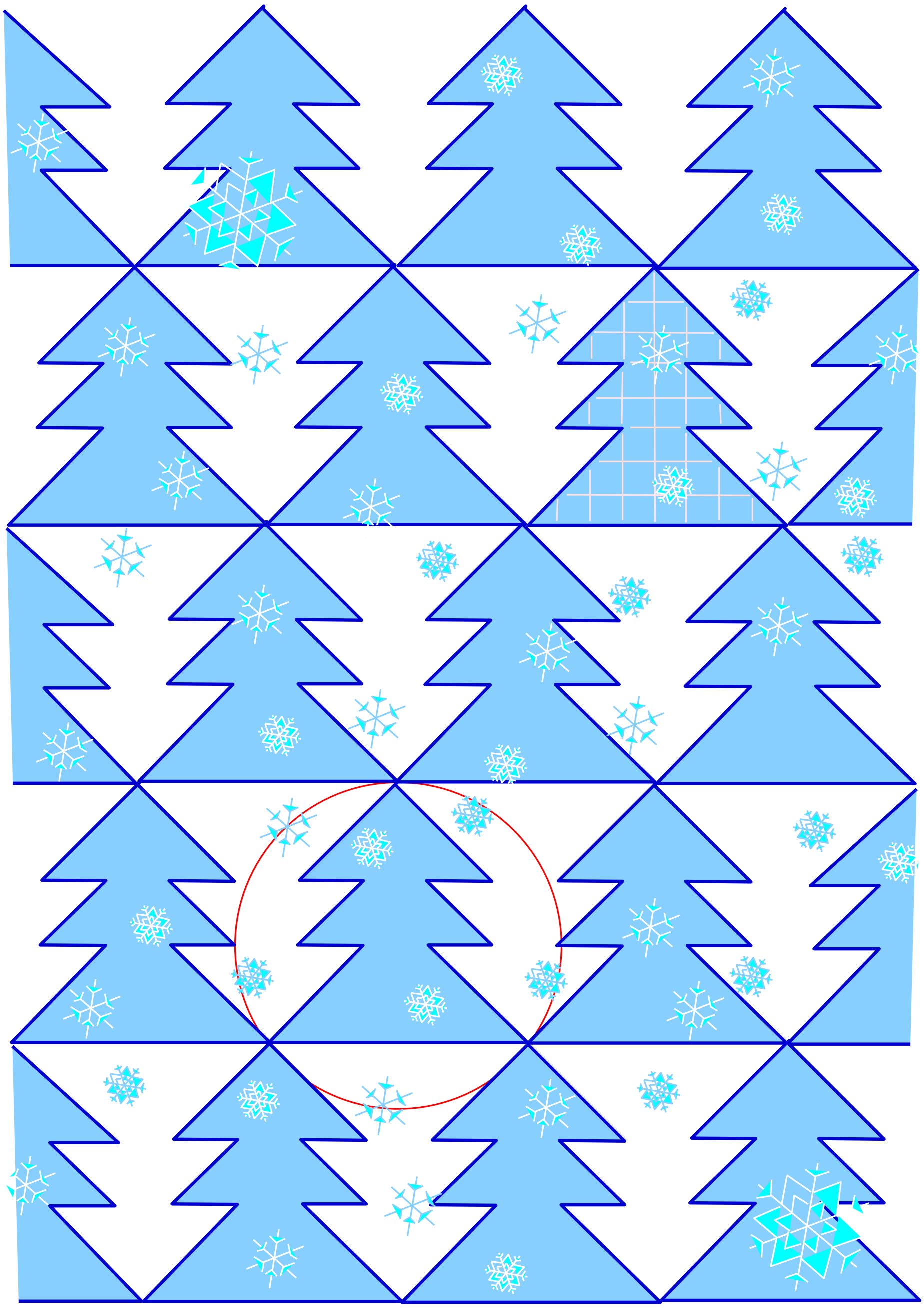
(a) Where should Alice place the point to get as much of the cake as possible?

(b) Now, modify the rules so that Alice gets the **bigger** part. Show that then Bob can always get exactly a half (if he could guess the right line l).

(c) In the settings of (b), can you propose an explicit algorithm allowing Bob to construct a line l which cuts the cake almost into two halves (i.e. as close to the two halves as Bob wants)?

(d) The same questions if the cake is a spherical triangle. (**Warning:** this question is difficult!)





Merry Christmas
and a Happy New Year!