

Spherical, Euclidean and hyperbolic geometries

	Spherical	Euclidean	hyperbolic
Lines	intersecting (2 pts of intersection)	intersecting (1 pt) parallel	intersecting (1 pt of intersection) parallel (1 pt on $\partial\mathbb{H}^2$) ultraparallel
orientation-preserving isometries	rotation	rotation translation	elliptic parabolic hyperbolic
congruence of triangles: AAA	+	-	+
Angle sum of a triangle	$> \pi$	$= \pi$	$< \pi$
Area of a triangle	$(\alpha + \beta + \gamma) - \pi$		$\pi - (\alpha + \beta + \gamma)$
Pythagorean thm	$\cos c = \cos a \cos b$	$c^2 = a^2 + b^2$	$\cosh c = \cosh a \cosh b$
Law of sines	$\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$	$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$	$\frac{\sinh a}{\sin \alpha} = \frac{\sinh b}{\sin \beta} = \frac{\sinh c}{\sin \gamma}$
Law of cosines	$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$ $\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a$	$a^2 = b^2 + c^2 - 2bc \cos \alpha$	$\cosh a = \cosh b \cosh c - \sinh b \sinh c \cos \alpha$ $\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cosh a$
Circumference of a circle	$2\pi \sin R$	$2\pi R$	$2\pi \sinh R$
Area of a disc	$4\pi \sin^2(\frac{r}{2})$	πR^2	$4\pi \sinh^2(\frac{r}{2})$