

Homework 9-10

- 9.1 Show that removing a small disc from a projective plane we get a Möbius band.
- 9.2 Removing a line from \mathbb{R}^2 or from S^2 one gets a space with two connected components. Show that $\mathbb{R}P^2 \setminus \mathbb{R}P^1$ is a connected space.
- 9.3. Let the Möbius band \mathbf{M} is obtained by gluing along the vertical sides of the square with vertices $(\pm 1, \pm 1)$. Let m be a midline of the Möbius band \mathbf{M} (obtained from the segment of the line $y = 0$).
- (a) what is $\mathbf{M} \setminus m$?
- (b) Let l be the closed line obtained from $y = 1/2$ and $y = -1/2$. What is $\mathbf{M} \setminus l$?
- 9.4. Let \mathbf{C} be a conic $x^2 + y^2 = z^2$. What kind of space is $\mathbb{R}P^2 \setminus \mathbf{C}$?
- 9.5. Given a point P inside a circle and a chord AB through the point P , let M_{AB} denote the intersection point of the two lines tangent to the circle at A and B . Show that M_{AB} runs over some line as A runs over the circle.
- 9.6. Let \mathbf{C} be a conic $x^2 + y^2 = z^2$. A triangle in $\mathbb{R}P^2$ is self-polar (with respect to \mathbf{C}) if its sides are polar to its vertices (not necessarily the opposite ones).
- (a) Construct a self-polar triangle with two vertices on \mathbf{C} .
- (b) Does there exist a self-polar triangle with exactly one vertex on \mathbf{C} ?
- (c) Show that there exists a self-polar triangle having no vertex on \mathbf{C} .
Hint: it may have some vertices at infinity.
- 9.7. (a) Formulate the theorem dual to Desargues' theorem.
- (b) Draw an example. (Hint: send the line s to the line at infinity).
- (c) Can you prove this theorem?
- 10.1. (a) How many non-intersecting lines can you draw in the Klein model of hyperbolic plane?
- (b) The same question, but no other line should intersect more than two lines of your family.
- 10.2. Show that any two lines on hyperbolic plane either intersect inside the hyperbolic plane, or intersect on its boundary, or have a unique common perpendicular.
- 10.3. (Right-angled polygons on hyperbolic plane)
- (a) Show that a hyperbolic triangle can not have more than 1 right angle.
- (b) Show that there are no hyperbolic rectangles (i.e. quadrilaterals with 4 right angles).
- (c) In the Klein model, construct a hyperbolic pentagon with 5 right angles.

References:

1. Lectures 17 and 18 follow Lecture IV of Prasolov's book.
2. Lecture 19 follows the section on Pappus' and Desargues' theorems in Chapter 3 of
 - V. V. Prasolov, V. M. Tikhomirov, *Geometry*, American Maths Soc., 2001.
3. Material of Lecture 20 (topology of projective plane and polarity on projective plane) may be found in Part II of
 - E. Rees, *Notes on Geometry*, Universitext, Springer, 2004.
(the book is available on DUO in Teaching Materials).