

Formula sheet

Sine and cosine laws:

	sine law	cosine laws
S^2	$\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$	$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$ $\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a$
\mathbb{E}^2	$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$	$a^2 = b^2 + c^2 - 2bc \cos \alpha$
\mathbb{H}^2	$\frac{\sinh a}{\sin \alpha} = \frac{\sinh b}{\sin \beta} = \frac{\sinh c}{\sin \gamma}$	$\cosh a = \cosh b \cosh c - \sinh b \sinh c \cos \alpha$ $\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cosh a$

Circles:

	S^2	\mathbb{E}^2	\mathbb{H}^2
Circumference of a circle	$2\pi \sin R$	$2\pi R$	$2\pi \sinh R$
Area of a disc	$4\pi \sin^2(\frac{R}{2})$	πR^2	$4\pi \sinh^2(\frac{R}{2})$

Angle of parallelism in hyperbolic geometry:

For a point on distance a from the line, the angle of parallelism φ satisfies

$$\sin \varphi = \frac{1}{\cosh a}$$

Distance formula in the upper half-plane model of hyperbolic geometry:

$$\cosh d(u, v) = 1 + \frac{|u - v|^2}{2\text{Im}(u)\text{Im}(v)}$$

Distance formula in the hyperboloid model of hyperbolic geometry:

For $u, v \in \mathbb{R}^{2,1}$, let $Q = \left| \frac{\langle u, v \rangle^2}{\langle u, u \rangle \langle v, v \rangle} \right|$. Then

if $\langle u, u \rangle < 0, \langle v, v \rangle < 0$ then $Q = \cosh^2 d(pt, pt)$

if $\langle u, u \rangle < 0, \langle v, v \rangle > 0$ then $Q = \sinh^2 d(pt, line)$

if $\langle u, u \rangle > 0, \langle v, v \rangle > 0$ then $Q < 1 \Rightarrow$ intersecting lines, $Q = \cos^2 \alpha$;
 $Q = 1 \Rightarrow$ parallel lines;
 $Q > 1 \Rightarrow$ ultraparallel lines, $Q = \cosh^2 d(line, line)$