

Assignment 13-14
Starred problems due on Friday, 19 February

- 13.1. Draw in each of the two conformal models (Poincaré disc and upper half-plane):
- (a) two intersecting lines;
 - (b) two parallel lines;
 - (c) two ultra-parallel lines;
 - (d) infinitely many disjoint (hyperbolic) half-planes;
 - (e) a circle tangent to a line.
- 13.2. In the upper half-plane model draw
- (a) a (hyperbolic) line through the points i and $i + 2$;
 - (b) a (hyp.) line through $i + 1$ orthogonal to the (hyp.) line represented by the ray $\{ki \mid k > 0\}$;
 - (c) a (hyperbolic) circle centred at i (just sketch it, no formula needed!);
 - (d) a triangle with all three vertices at the absolute (such a triangle is called *ideal*).
- 13.3. Prove SSS, ASA and SAS theorems of congruence for triangles on hyperbolic plane.
- 13.4. Let ABC be a triangle. Let $B_1 \in AB$ and $C_1 \in AC$ be two points such that $\angle AB_1C_1 = \angle ABC$. Show that $\angle AC_1B_1 > \angle ACB$.
- 13.5. Show that there is no “rectangle” in hyperbolic geometry (i.e. no quadrilateral has four right angles).
- 13.6. (*) Given an acute-angled polygon P (i.e. a polygon with all angles smaller or equal to $\pi/2$) and lines m and l containing two disjoint sides of P , show that l and m are ultra-parallel.
- 14.7. Given α, β, γ such that $\alpha + \beta + \gamma < \pi$, show that there exists a hyperbolic triangle with angles α, β, γ .
- 14.8. Show that there exists a hyperbolic pentagon with five right angles.
- 14.9. (*) An *ideal* triangle is a hyperbolic triangle with all three vertices on the absolute.
- (a) Show that all ideal triangles are congruent.
 - (b) Show that the altitudes of an ideal triangle are concurrent.
 - (c) Show that an ideal triangle has an inscribed circle.
- 14.10. (*) We know that an isometry fixing 3 points of the absolute is the identity map. How many isometries fix two points of the absolute? Classify the isometries fixing 0 and ∞ in the upper half-plane model.
- 14.11.
 - (a) Show that the group of isometries of hyperbolic plane is generated by reflections.
 - (b) How many reflections do you need to map a triangle ABC to a congruent triangle $A'B'C'$?
- 14.12. (*)
- (a) Does there exist a regular triangle on hyperbolic plane?
 - (b) Does there exist a right-angled regular polygon on hyperbolic plane? How many edges does it have (if exists)?
- 14.13.
 - (a) Show that the angle bisectors in a hyperbolic triangle are concurrent.
 - (b) Show that every hyperbolic triangle has an inscribed circle.
 - (c) Does every hyperbolic triangle have a circumscribed circle?

References:

Lectures 25-28 (Conformal models of hyperbolic plane) are based on Lectures VI and VII in Prasolov's book. Alternatively, see pp.95–104 in Section 5.2 of Prasolov, Tikhomirov.