

## Formula sheet

Sine and cosine laws:

	sine law	cosine laws
$S^2$	$\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$	$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$ $\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a$
$\mathbb{E}^2$	$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$	$a^2 = b^2 + c^2 - 2bc \cos \alpha$
$\mathbb{H}^2$	$\frac{\sinh a}{\sin \alpha} = \frac{\sinh b}{\sin \beta} = \frac{\sinh c}{\sin \gamma}$	$\cosh a = \cosh b \cosh c - \sinh b \sinh c \cos \alpha$ $\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cosh a$

Circles:

	$S^2$	$\mathbb{E}^2$	$\mathbb{H}^2$
Circumference of a circle	$2\pi \sin R$	$2\pi R$	$2\pi \sinh R$
Area of a disc	$4\pi \sin^2(\frac{R}{2})$	$\pi R^2$	$4\pi \sinh^2(\frac{R}{2})$

Angle of parallelism in hyperbolic geometry:

For a point on distance  $a$  from the line, the angle of parallelism  $\varphi$  satisfies

$$\sin \varphi = \frac{1}{\cosh a}$$

Distance formula in the upper half-plane model of hyperbolic geometry:

$$\cosh d(u, v) = 1 + \frac{|u - v|^2}{2\operatorname{Im}(u)\operatorname{Im}(v)}$$

Distance formula in the hyperboloid model of hyperbolic geometry:

For  $u, v \in \mathbb{R}^{2,1}$ , let  $Q = \left| \frac{\langle u, v \rangle^2}{\langle u, u \rangle \langle v, v \rangle} \right|$ . Then

if  $\langle u, u \rangle < 0, \langle v, v \rangle < 0$  then  $Q = \cosh^2 d(pt, pt)$

if  $\langle u, u \rangle < 0, \langle v, v \rangle > 0$  then  $Q = \sinh^2 d(pt, line)$

if  $\langle u, u \rangle > 0, \langle v, v \rangle > 0$  then  $Q < 1 \Rightarrow$  intersecting lines,  $Q = \cos^2 \alpha$ ;  
 $Q = 1 \Rightarrow$  parallel lines;  
 $Q > 1 \Rightarrow$  ultraparallel lines,  $Q = \cosh^2 d(line, line)$