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Feedback 1-2

Overall, it was an easy assignment and most of students did it very well.

- **Question 2.1:**
  - Useful ideas:
    - know the classification of isometries;
    - look at decompositions into reflections.
  - Typical mistakes:
    - some of you have not explained why to look only at rotations and translations (classification of isometries + orientation);
    - when demonstrating that translations are compositions of rotations, some students assumed that
      $$\textit{if } f(A) = A' \textit{ and } g(A) = A' \textit{ then } f = g,$$
      which is wrong, you need to check images of 3 points to derive that.
    - Strictly speaking, there are 5 types of isometries of $\mathbb{E}^2$, i.e. four non-trivial types and the identity. There is nothing to check for the identity, but it is not completely correct to write that there are exactly 4 types of isometries (I did not took any points for this, though).
  - A source of confusion:
    - Absence of clear understanding what does it mean when a group G is generated by a set $S \subset G$ of elements $\{s_i \in S\}$. Informally, it means that G is the smallest group containing all elements of S (in particular, containing all their inverses). Formally:

      Definition. A group G is generated by a set $S \subset G$ of elements $\{s_i \in S\}$ if for every $g \in G$ there exist $n \in \mathbb{Z}$ and $s_{i_1}, s_{i_2}, \dots, s_{i_n} \in S$ such that $g = s_{i_n}^{\pm 1} \circ \cdots \circ s_{i_2}^{\pm 1} \circ s_{i_1}^{\pm 1}$. The elements of S in this case are called generators.

      In particular, **elements** of $Isom^+(\mathbb{E}^2)$ are identity, rotations and translations.
      Generators for the group may be chosen in many ways. Here are some options:
      - all elements of $Isom^+(\mathbb{E}^2)$;
      - all rotations;
      - rotations preserving the origin and translations.
- **Question 2.4:**
  - Useful idea:
    - you can use the classification of isometries without reproving it! (not need to derive from the fact that every isometry is a composition of 3 reflections).
  - Typical mistakes:
    - Some of you forgot that a rotation may be by an angle $\alpha\pi$ where α is irrational.
    - Others forgot that there are rational numbers other than $1/n$, $n \in \mathbb{Z}$.
- **Question 2.7:**
  - Comments:
    - It is straightforward to figure out *what* to compute in part (a). To make this computation shorter, compute in the vector form, not in coordinates! See solutions.
    - The idea of this question was to obtain two different solutions:
      (a)-(c): prove f is isometry by a computation,
      then use the fixed points to see it is a reflection;
      (d): see that f is a reflection directly from geometry.
      (First solution is straightforward, second is not).
    - To demonstrate the geometric solution, it is very useful to draw a diagram.