Anna Felikson, Durham University **Geometry**, 24.11.2025

Feedback 5-6

First 3 questions of this assignment was straightforward and most of students did it very well, while question 6.5 was a harder one (not many figured out how to do (b)).

- **Question 5.2:**
  - This question follows very quickly from the idea of locus of points...
  - More advanced students also understand that one should show that the the perpendicular bisector on a sphere is still the locus of points ... (as the proof is exactly the same as for $\mathbb{E}^2$, I have not really required this for the correct solution). Similarly, with the angle bisector.
  - Several students thought that "the perpendicular bisector for AB is the locus of points on the same distance from A and B" is the definition of the perpendicular bisector. No! This is a property! Not a definition! (By definition, the perpendicular bisector is the line orthogonal to AB and passing thorugh the midpoint of AB).
- **Question 5.4:**
  - Many solutions said: "consider a triangle with three right angles, then ..." Why does this triangle exist? The answer may be given in coordinates or constructed using the notion of polarity.
  - It was very tempting to use Bipolar Theorem and to get a relation for sides/angles of a self-polar triangle. However, it only gives $a = \pi - \alpha$, which is not clear how to use.
  - So, after all it was easier just to construct the triangle point by point (uniquely up to isometry). One can also collect all right angles (or sides) while doing the construction and apply AAA or SSS rule of congruence of triangles.
- **Question 5.8:**
  - It is clear one needs to apply a formula here and almost clear which one. The problem is that one needs to compute correctly:
    - use a circle to check the values of sine and cosine!
    - pay attention to the signs: cosine of an obtuse angle should be negative!
    - you don't need the calculator for the Geometry course, it does not really help to arrive to a good solution...
    - every year a couple of very strong students looses marks for doing silly mistakes working with sines and cosines. So, check your solutions, it is a dangerous point.
- **Question 6.5:**
  - From Hints, it is clear that one needs to use a projection, and luckily, in (a) the same projection as in 4.4 works.
  - But one needs to show that the projection takes medians to medians (which is not difficult).
  - However, for altitudes the same projection does not work (does not always take altitudes to altitudes)!

    To see this, consider a triangle ABC with $AC \neq BC$. Let CH be the spherical altitude, and let C_t be a point moving along CH from H (at $t = 0$) to C (at $t = 1$). Then each triangle ABC_t has an altitude on CH, and for triangles with t small enough it is easy to see that the projection of its altitude on the plane ABC_t is very close to the line OH, which means it is not perpendicular to AB.
  - This shows how important it was to check that altitudes are projected to altitudes...
  - However, in some solutions it was checked and proved! The typical mistake was based on the reasoning like the following:

    For lines a, b and planes α, β, prove that $a \in \alpha$, $b \in \beta$. Given $\alpha \perp \beta$, conclude that $a \perp b$.