

# Riemannian Geometry

## Hints 9–10

1. Look at the corresponding differential equation.
2. (\*) Follow the same plan as for the First Variation formula of Length:
  - (a) Differentiate  $E(s)$  at 0 using Riemannian property of the Levi-Civita connection. Then apply the Symmetry Lemma and use Riemannian property again.
  - (b-d) straightforward.
  - (e-f) as in the proof of Theorem 5.9 and Corollary 5.10.
4. Consider  $F(s, t) = \exp_{c(t)}(sX(t))$ .
5. (a) Show that  $\left. \frac{\partial}{\partial x_i} \right|_p = v_i$ .
  - (b) Write the equation of geodesic  $\frac{D}{dt}c' = 0$  in the coordinates.
  - (c) Evaluate the equations obtained at (b) at  $t = 0$ ; choose vectors  $w = e_i$  and  $w = e_i + e_j$  to find  $\Gamma_{ii}^k(p)$  and  $\Gamma_{ij}^k(p)$ .