

Riemannian Geometry

Remark on Homework 2

Some students told me that it is not clear what the sign “ \sim ” means in the question 3.

This sign is called “equivalence”, and when it is written X/\sim or $x \rightarrow x/\sim$ this means that we “identify the equivalent points of X ” (consider all equivalent points as one point of the new space).

In particular, when we construct a projective space from a sphere, we identify each point of the sphere with diametrically opposite one.

Example: in 1-dimensional case we get a half of a circle with the endpoints identified — hence, $\mathbf{R}P^1$ is a circle.

In two dimensions the same construction gives a new surface called $\mathbf{R}P^2$. More precisely, it is a hemisphere with the opposite boundary points identified. After the identification you get a surface that is locally looks still like a sphere, but globally is different. In particular, it is a one-sided surface and you can not introduce an orientation there.

It is impossible to embed this surface in \mathbf{R}^3 without self-crossings. You can google “projective plane” to find some models and pictures describing $\mathbf{R}P^2$.