MATH4171
Riemannian Geometry

Time and place: Tuesday and Friday, 12:00 CM107
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Topology prerequisites

We will use the notion of *open set* and *topology* (some of you may recall it from Topology III, others, hopefully, have seen some examples in Differential Geometry III).

1. Open set in a metric space. Let M be a metric space, i. e. a set with a distance $d(x, y)$ defined for all points $x, y \in M$ (and satisfying 4 axioms of metric: non-negativity, identity, symmetry, triangle inequality).

An **open ball** $B_r(x)$ of radius r around a point $x \in M$ is the set of all points in M such that

$$B_r(x) = \{y \in M \mid d(x, y) < r\}.$$

An **open subset** $U \in M$ is a subset U such that for each point $x \in U$ there exists an open ball $B_r(x)$ also contained *wholly* in U:

$$U \in M \textit{ is open } \iff (\forall x \in U \quad \exists r > 0 \textit{ such that } B_r(x) \subset U).$$

Example: The set $\{\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n \mid x_i > 0 \quad \forall i = 1, \dots, n\}$ is open in $\mathbb{R}^n$ as it contains each point $\mathbf{x}^0$ together with the open ball $B_r(\mathbf{x}^0)$ where $r = \frac{1}{2}\min\{x_1^0, \dots, x_n^0\}$.

2. Open set in a topological space. More generally, starting with a set X, a **topology** on X is just a collection of subsets of X that are called, again, **open sets**. These open sets have to satisfy the following axioms:

- the empty set and X itself are open;
- any union of open sets is open;
- the intersection of any finite number of open sets is open.

If U is an open set containing a point x, we call U an **open neighbourhood** of x.

Example: the set of all open balls on $\mathbb{R}^n$ satisfies the three axioms above and so defines a topology on $\mathbb{R}^n$. When we talk about *the* topology on $\mathbb{R}^n$, this is the topology we mean.