

Riemannian Geometry, Hints 2

- 2.1** (b): show that $GL_n(\mathbb{R})$ is open in $M_n(\mathbb{R})$ and use the result of (a).
- 2.2** (b): write $A^t A$ in terms of elements of A , then define a map $f : M_2(\mathbb{R}) \rightarrow \mathbb{R}^3$ which takes A to the triple of different elements of $A^t A$ and consider $O_2(\mathbb{R})$ as a preimage of some regular value of f .
- 2.3** This is very similar to 2.2.
- 2.4** Differentiate $\lambda \mapsto f(\lambda x_1, \lambda x_2, \dots, \lambda x_k)$ with respect to λ and use homogeneity. Then consider $\lambda = 1$.
- 2.5** The solution is straightforward.
- 2.6** Just use the definition of the directional derivative.