Riemannian Geometry, Hints 3

3.1 Write $f \circ \varphi^{-1}$ as $f \circ \psi^{-1} \circ \psi \circ \varphi^{-1}$ and apply the chain rule.

3.2 First, find $\varphi \circ \gamma(t)$ as a pair $(\gamma_1(t), \gamma_2(t)$.

3.3 (a) Use Implicit Function Theorem.

(b) Write the coordinates on $\mathbb{C}$ as $\alpha + i\beta$. For the basis vectors $\frac{\partial}{\partial b}, \frac{\partial}{\partial c}, \frac{\partial}{\partial d}$ of $T_{(1,0)}S^3$ consider some curves γ_b, γ_c and γ_d such that the directional derivatives along these curves coincide with $\frac{\partial}{\partial b}, \frac{\partial}{\partial c}, \frac{\partial}{\partial d}$. Then consider the images of these curves under the map π and write the directional derivatives along $\pi(\gamma_b)$, $\pi(\gamma_c)$ and $\pi(\gamma_d)$ in the basis $\langle \frac{\partial}{\partial \alpha}, \frac{\partial}{\partial \beta} \rangle$.

3.4 You may use that we have, componentwise, $(AB)'(s) = A'(s)B(s)+A(s)B'(s)$ for the product of any two matrix-valued curves, and $(A^t)'(s) = (A'(s))^t$.