Riemannian Geometry, Hints 9

9.1 (a) Use Riemannian property and Symmetry Lemma (following proof of Theorem 5.12).

(b) By definition of geodesic, $\frac{D}{dt}c' = 0$.

(c) If F is proper then $X(a) = 0$ and $X(b) = 0$.

(d) Combine (b) and (c).

(e) Assume the contrary (i.e. $\frac{D}{dt}c'(t_0) \neq 0$ for some t_0 and use it to construct a variational vector field for some proper variation F, so that $E'(0) \neq 0$ for F. (You may want to consider $X = \varphi(t)\frac{D}{dt}c'(t)$ for some smooth function $\varphi(t)$).

9.3 (a) First show by that you only need to check $T(fX, Y) = fT(X, Y)$, then use properties of the Lie bracket and affine connection.

(b) Similar to (a) (using properties of affine connection and definition of covariant derivative).

(c) Use the result of (b).