Reflection subgroups of Coxeter groups

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$H \subset G$ is a finite index reflection subgroup.

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- If $X = \mathbb{S}^n$, groups are classified. For subgroups see:

E. B. Dynkin, 1952 and F, 2002.

- If $X = \mathbb{E}^n$, groups are classified. For subgroups see:

Dyer, 1990; Cameron, Seidel, Tsaranov, 1994; F&T, 2005.

- If $X = \mathbb{H}^n$, groups are not classified. What about subgroups ?

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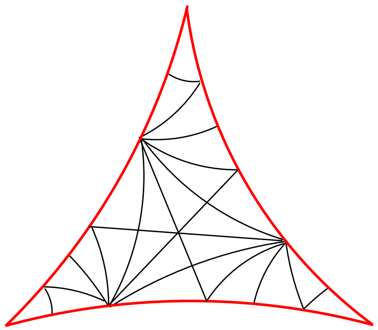

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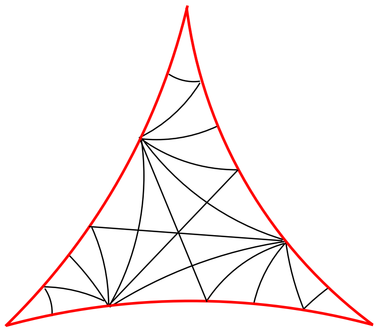

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quadrilateral	simplex or quadrilateral
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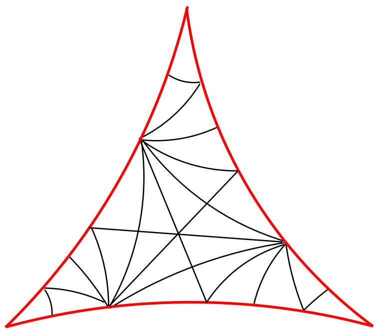

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Is the combinatorics of F always simpler than one of P?

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Claim: $|P| \geq |F|$.

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Theorem (F&T, '03). *Let F be a finite volume Coxeter polytope in $\mathbb{H}^n$ or $\mathbb{E}^n$ and P be a finite volume polytope bounded by mirrors of G_F. Then $|P| \geq |F|$.*

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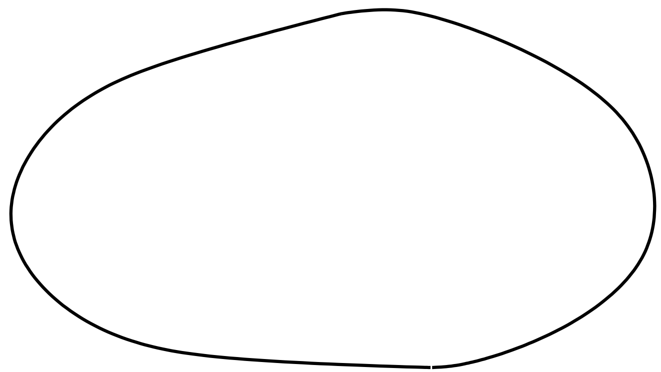

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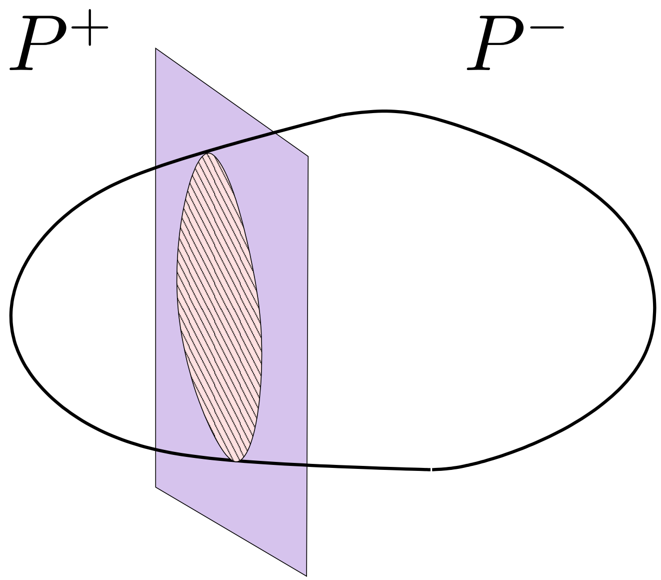

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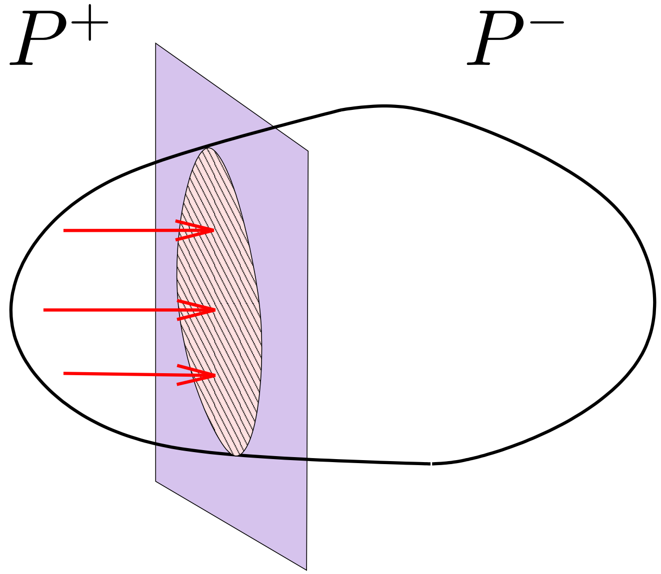

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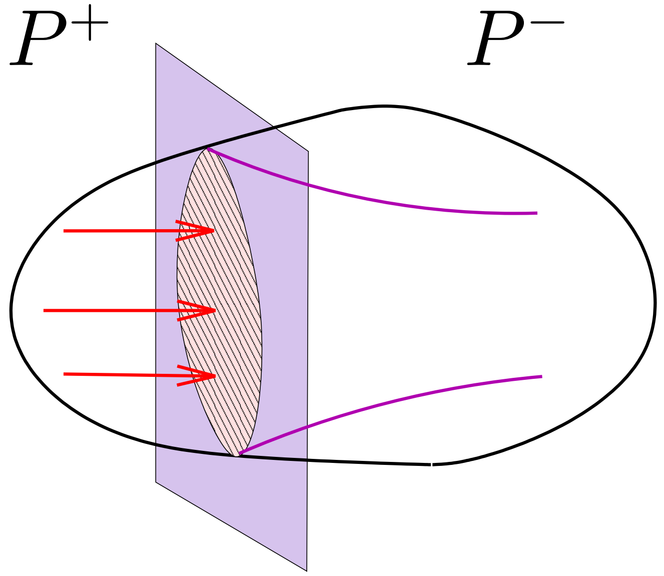

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Pair (P'_{min}, P_{min}) satisfies assumptions of Lemma. Contradiction.

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If G is a cocompact reflection group acting on $\mathbb{H}^n$ and $H \subset G$ is a finite index reflection subgroup then $\#(\text{refl. gen. } H) \geq \#(\text{refl. gen. } G)$.

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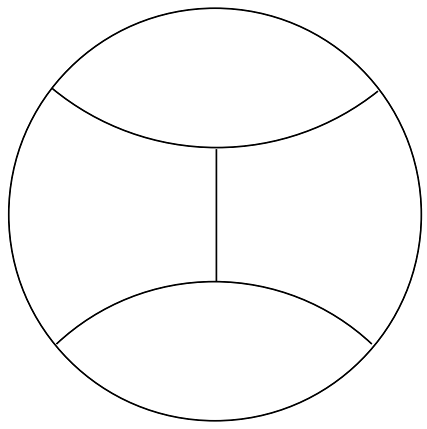

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Does it hold for arbitrary infinite indecomposable Coxeter group?

(G, S) is a Coxeter system if G is a group with a finite set of involutions $S = \{s_1, ..., s_n\}$ and a presentation

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Prop. *G cannot be gen. by fewer than n reflections.*

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  it is a convex hull C of a G-orbit of a suitable point p,

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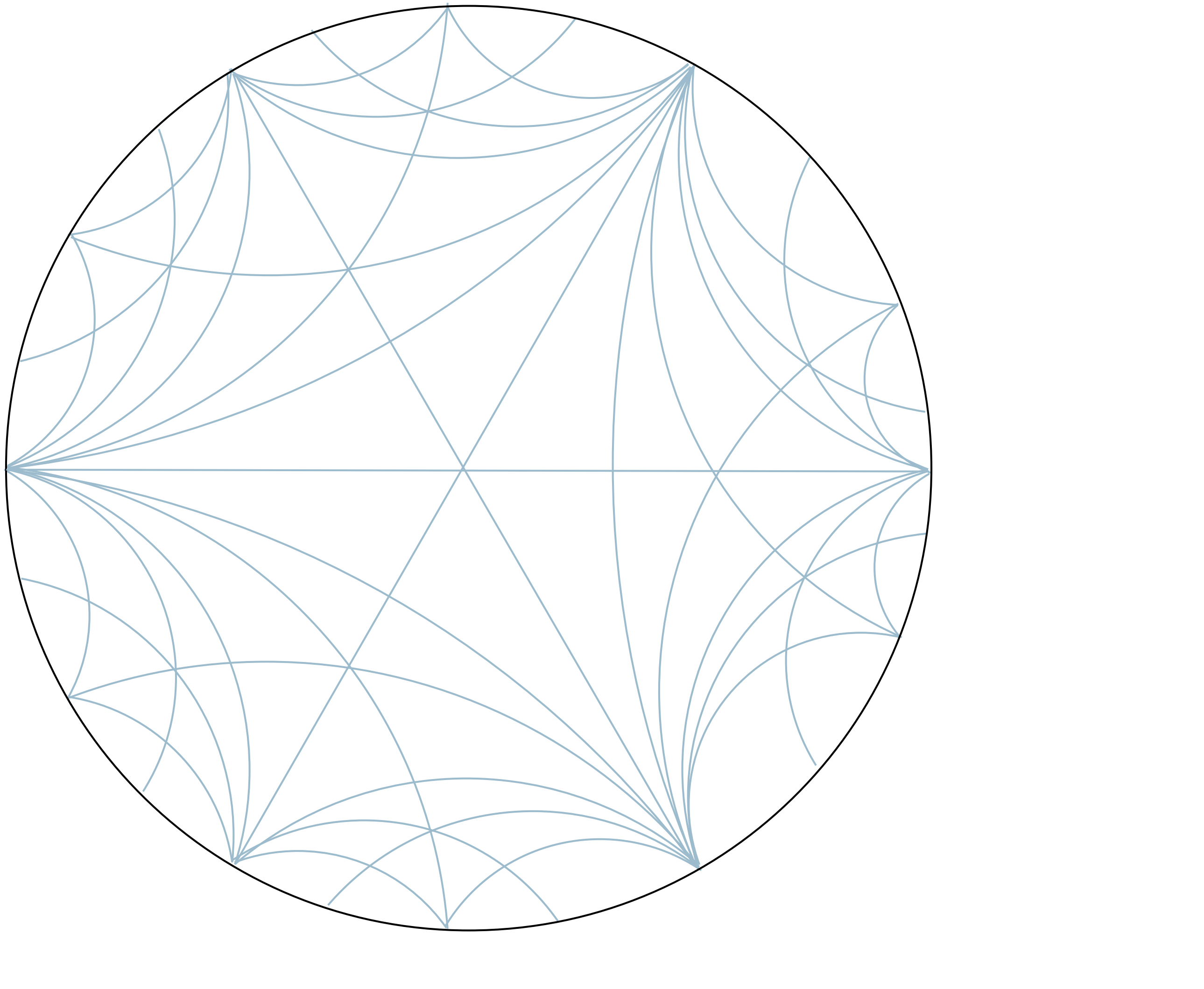
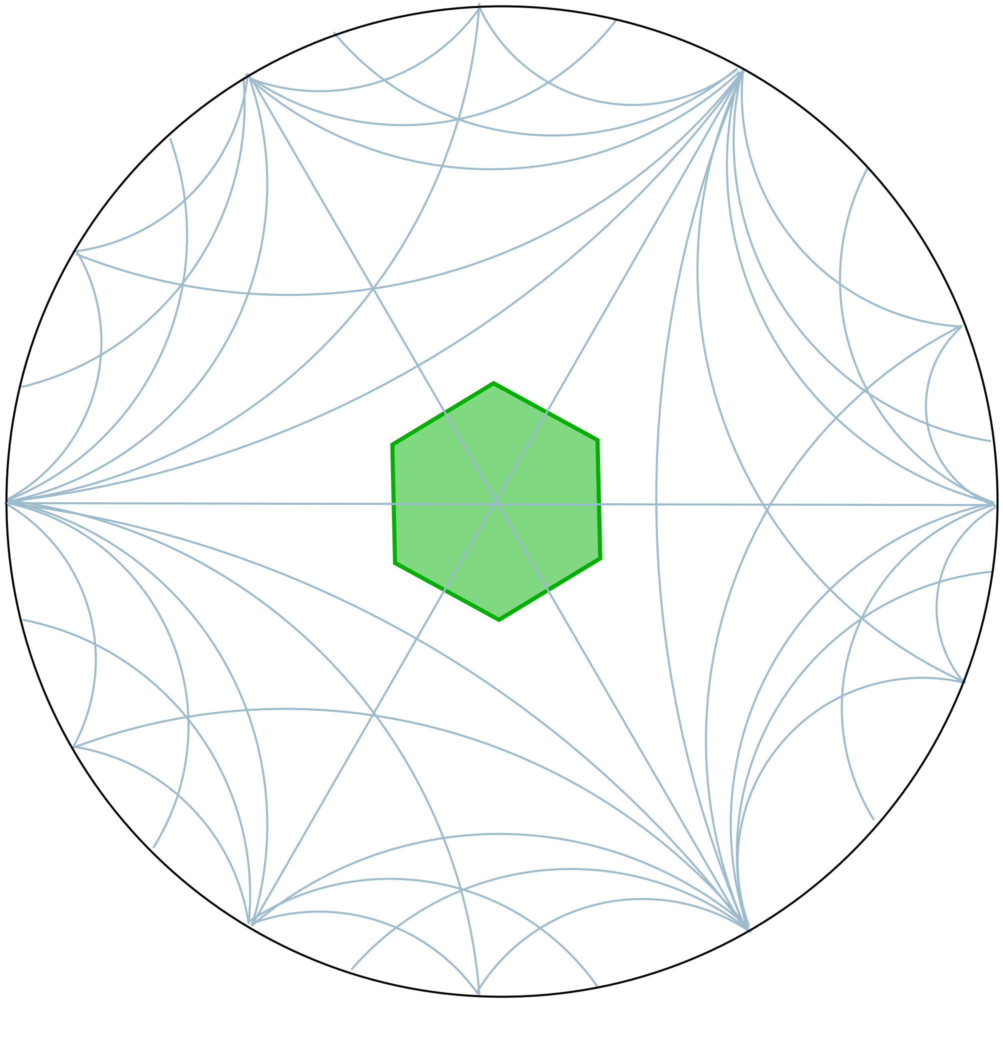
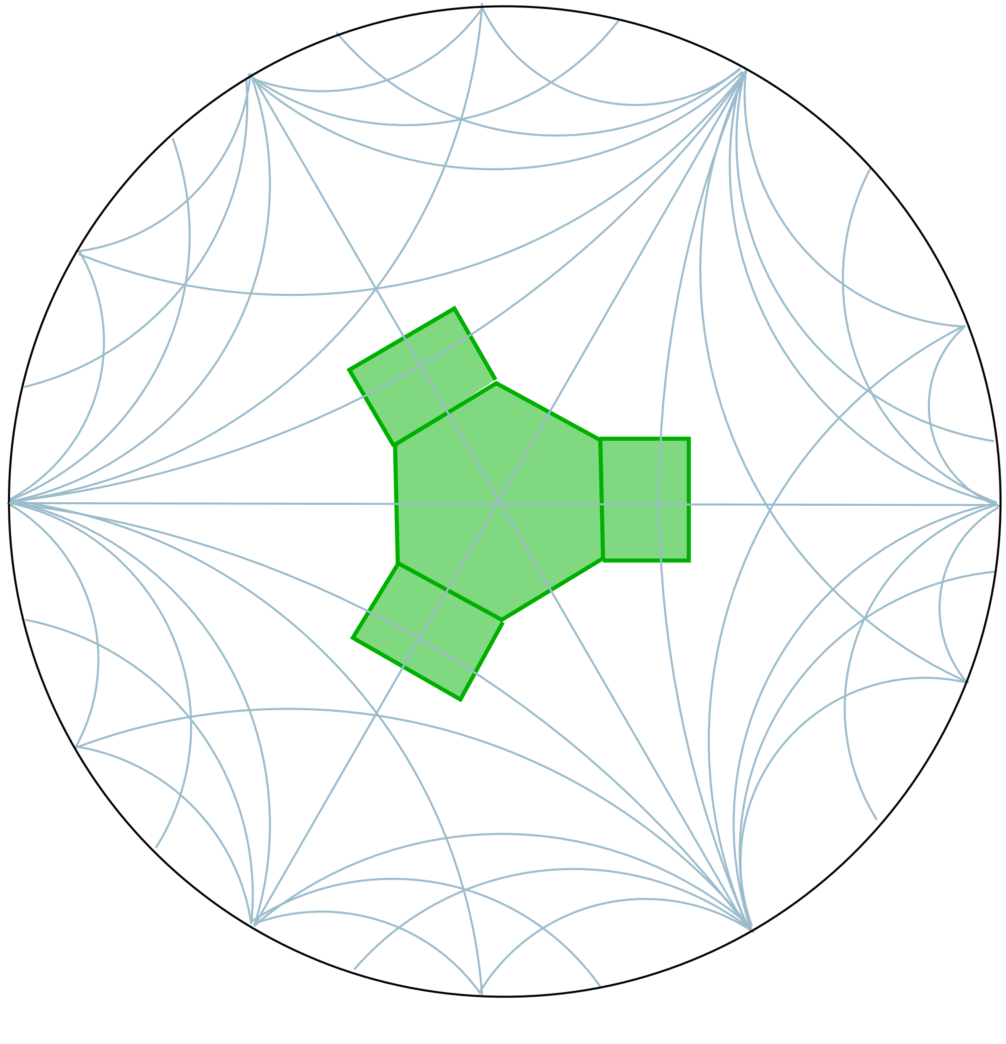
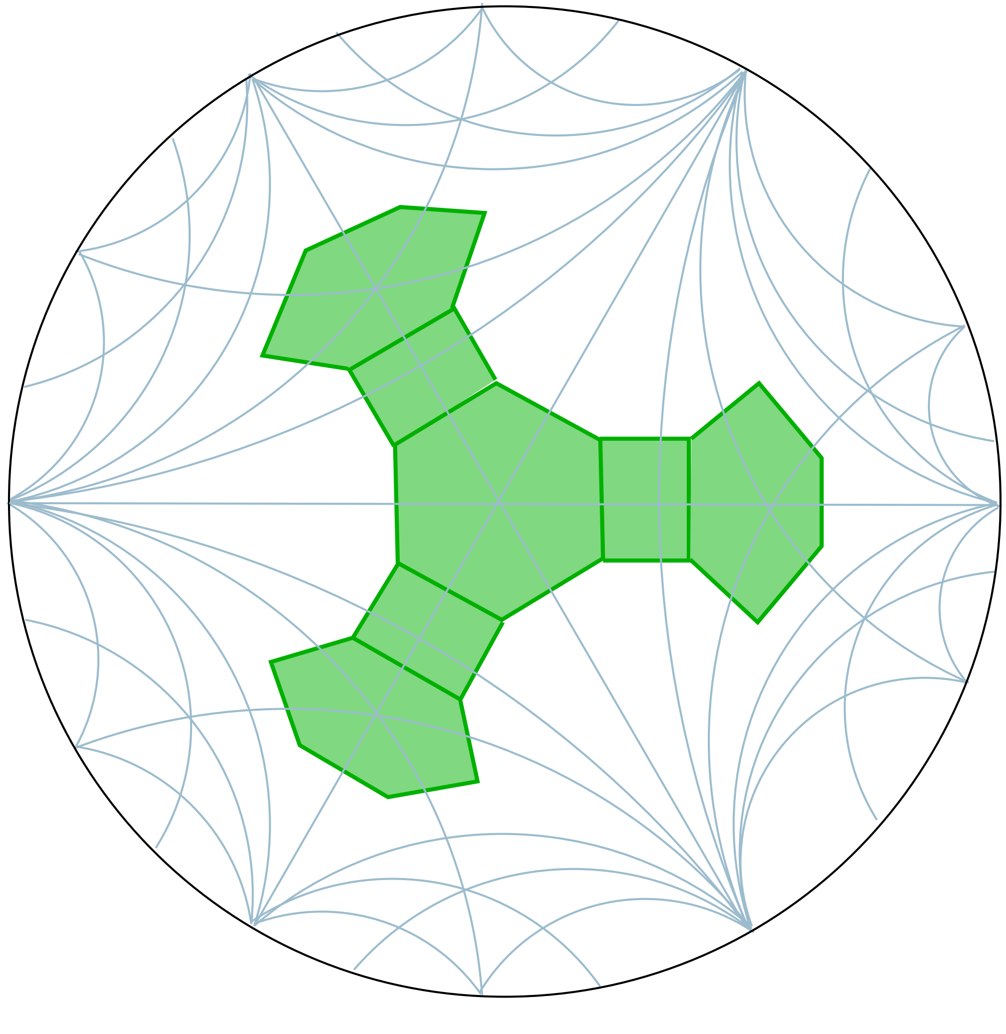
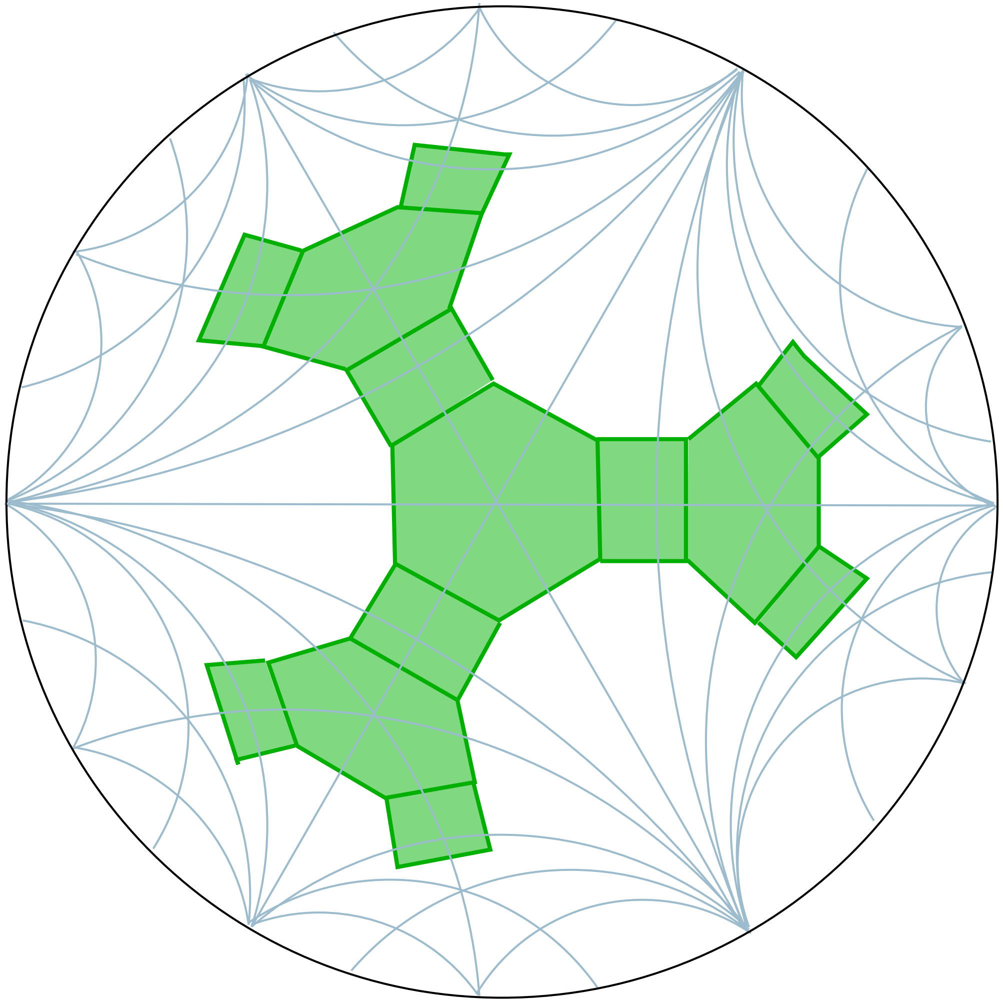
• For an infinite group $\Sigma(G, S)$ is built up of the Davis complexes of maximal finite subgroups (glued together along their faces corresponding to common subgroups).

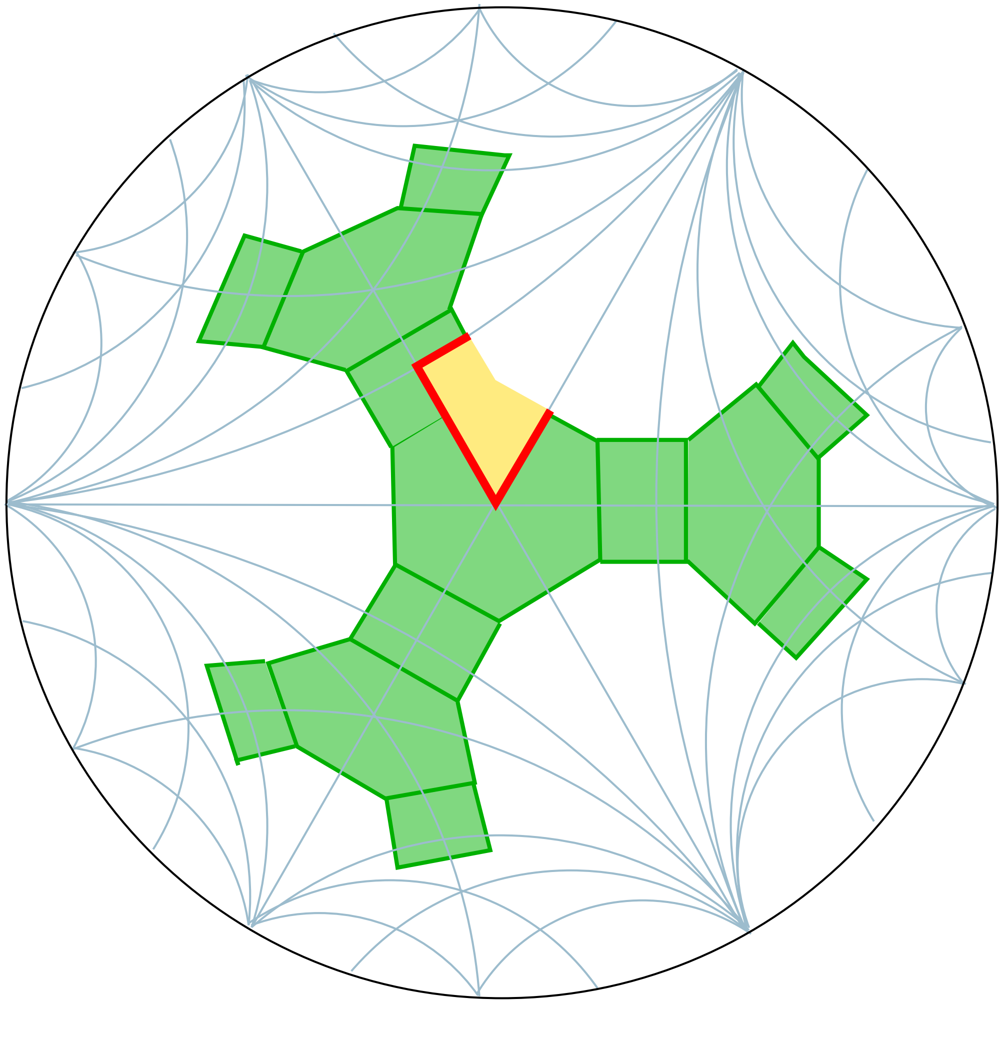












Any wall α decomposes $\Sigma(G, S)$ into two components α^+ and α^-.

A convex polytope is an intersection of finitely many halfspaces

$$P = \bigcap_{i=1}^{m} \alpha_i^+.$$

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G is decomposable if $S = S_1 \cup S_2$, where $s_i s_j = s_j s_i \quad \forall s_i \in S_1, s_j \in S_2$.

Otherwise, G is indecomposable.

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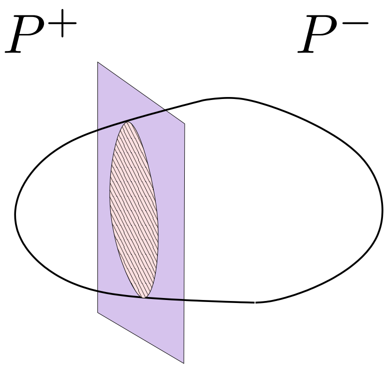

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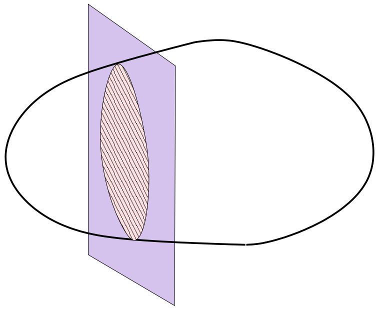

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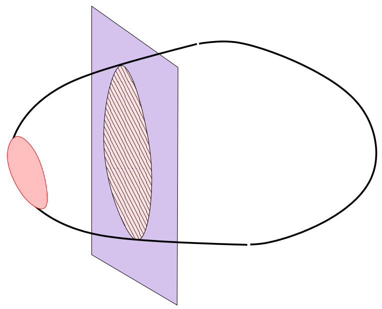

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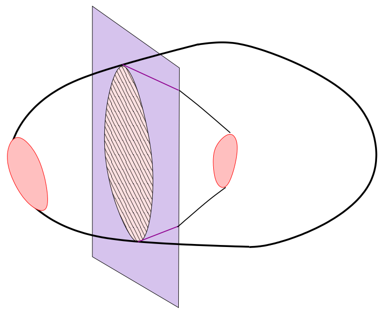

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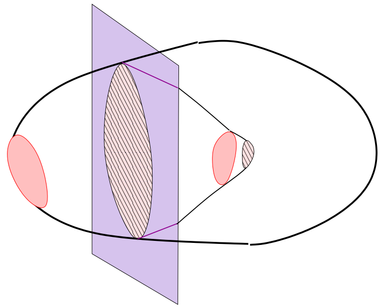

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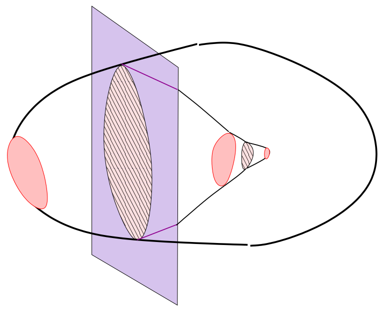

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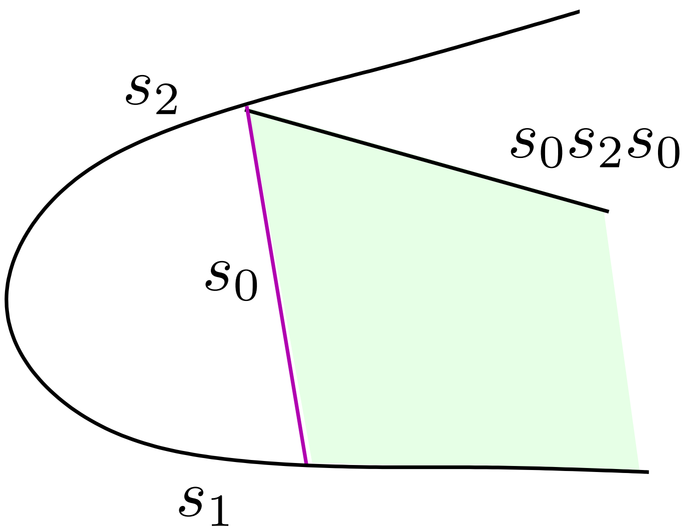

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Hence

$$\# \begin{pmatrix} \text{chambers of } s_0 H s_0 \\ \text{in } \langle s_1, s_0 s_2 s_0, \ldots, s_0 s_n s_0 \rangle \end{pmatrix} = \# \begin{pmatrix} \text{chambers of } s_0 H s_0 \\ \text{in } \langle s_0 s_2 s_0, \ldots, s_0 s_n s_0 \rangle \end{pmatrix}$$

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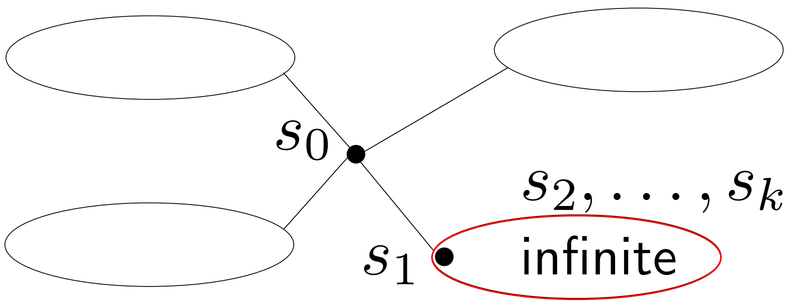

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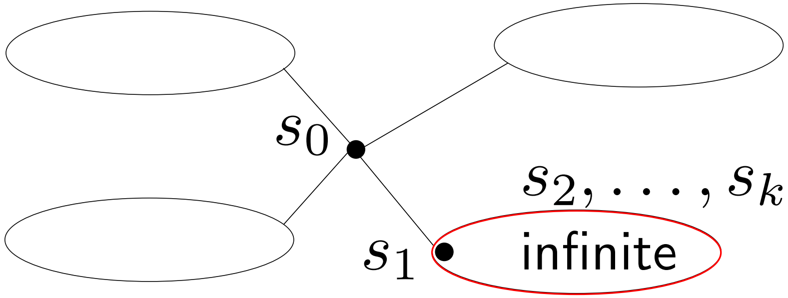

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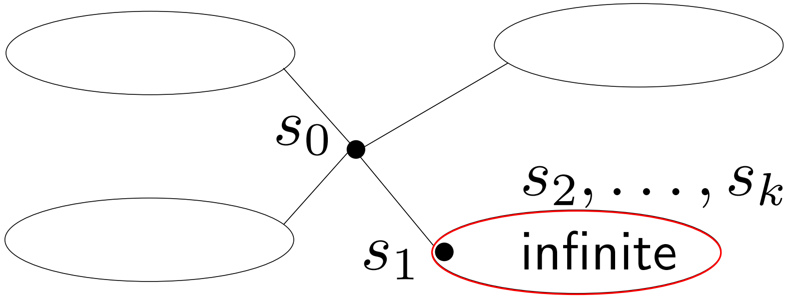

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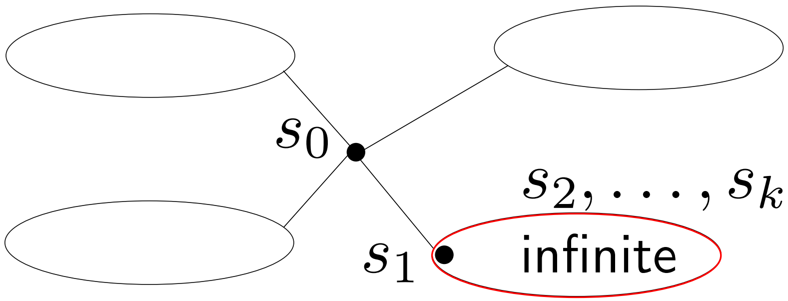

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$$\#\begin{pmatrix} \text{chambers of } s_0 K s_0 \\ \text{in } \langle s_1, s_0 s_2 s_0, \ldots, s_0 s_n s_0 \rangle \end{pmatrix} = \#\begin{pmatrix} \text{chambers of } s_0 K s_0 \\ \text{in } \langle s_0 s_2 s_0, \ldots, s_0 s_k s_0 \rangle \end{pmatrix} = \infty$$

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Let $P \subset \Sigma$ *be an acute-angled polytope, let* a *and* b *be facets of* P *and* α *and* β *be the walls containing* a *and* b *respectively.*
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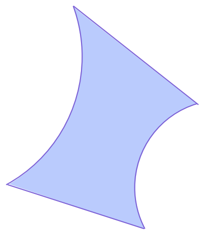

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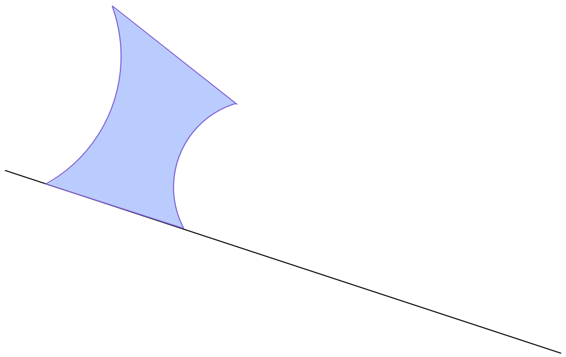

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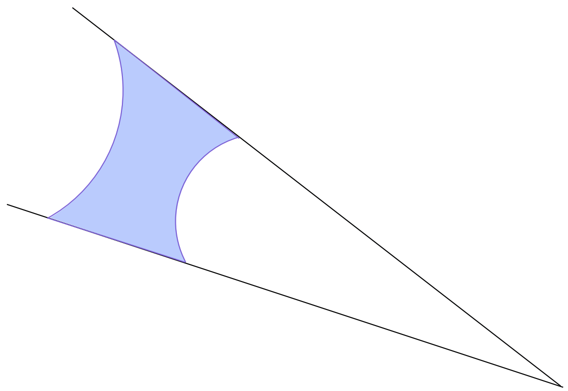

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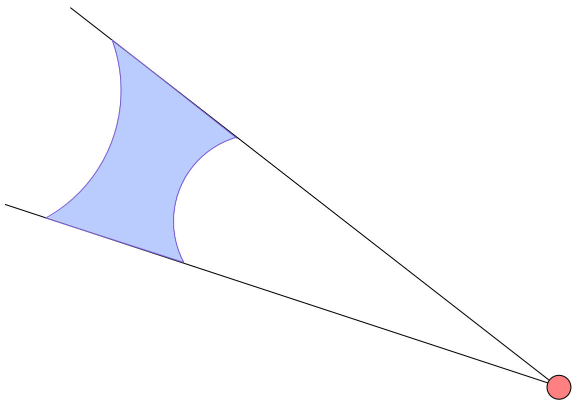

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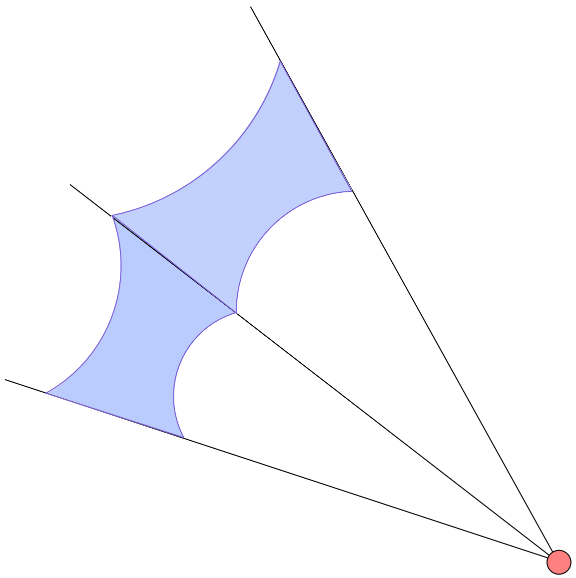

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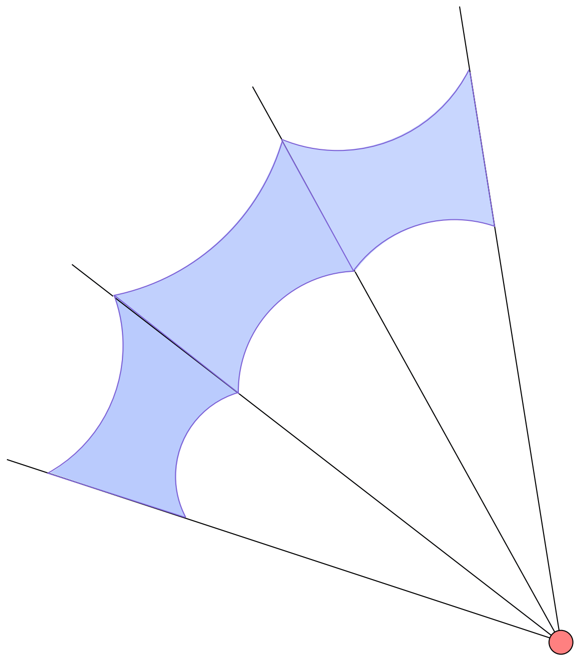

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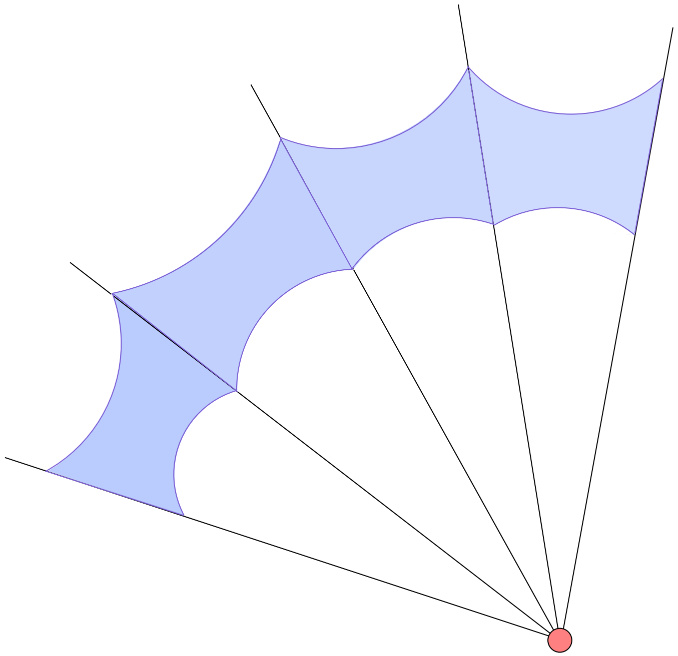

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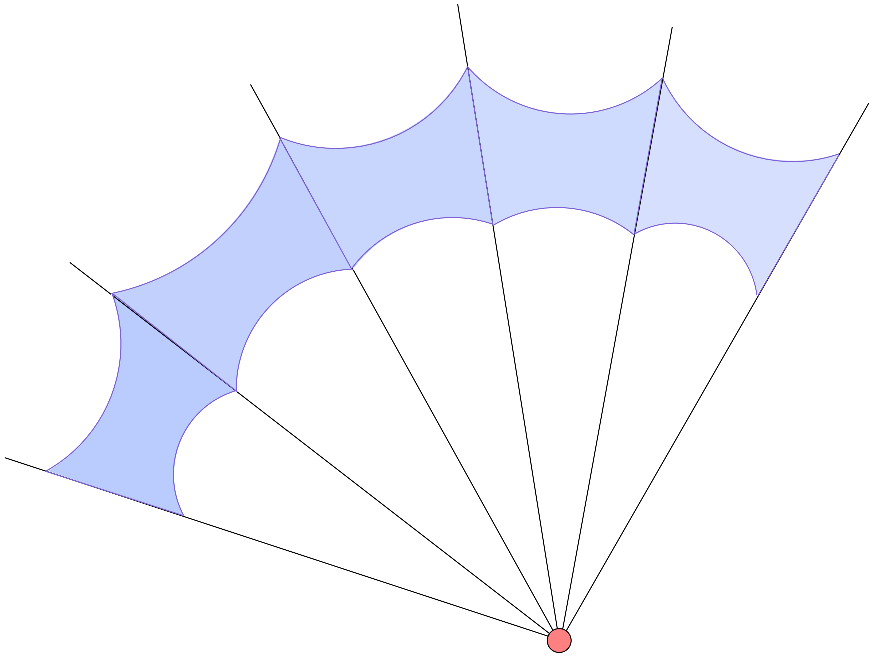

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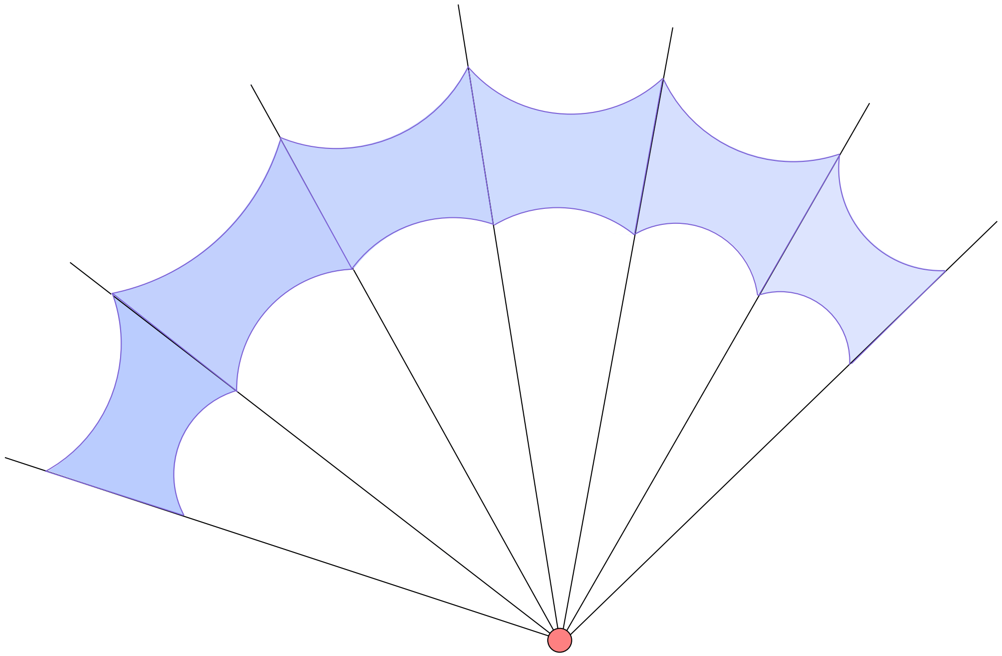

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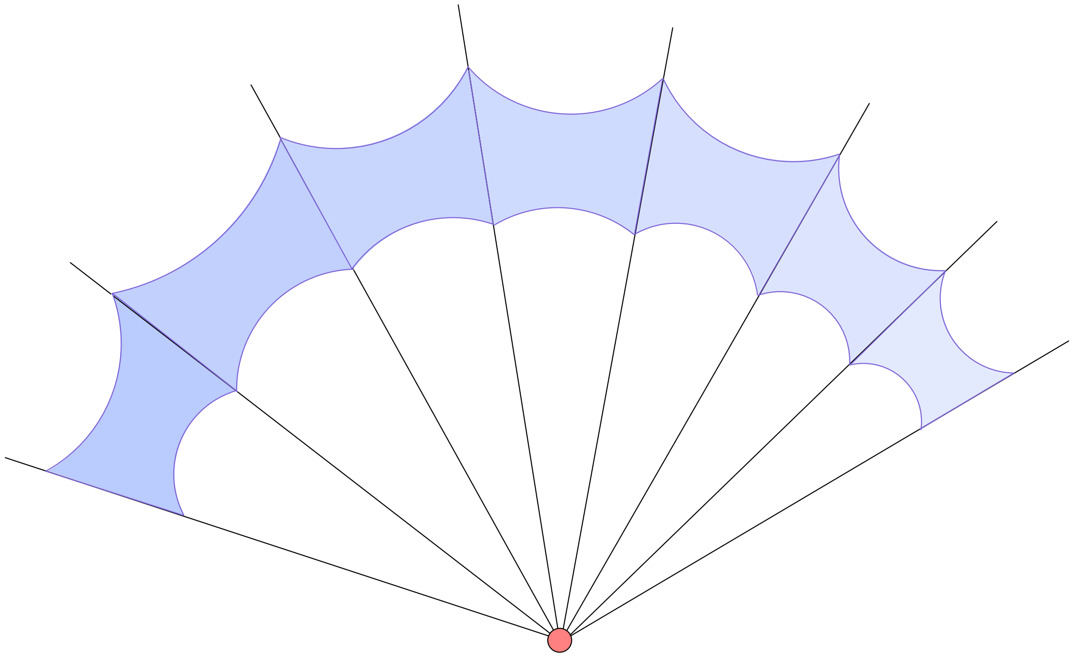

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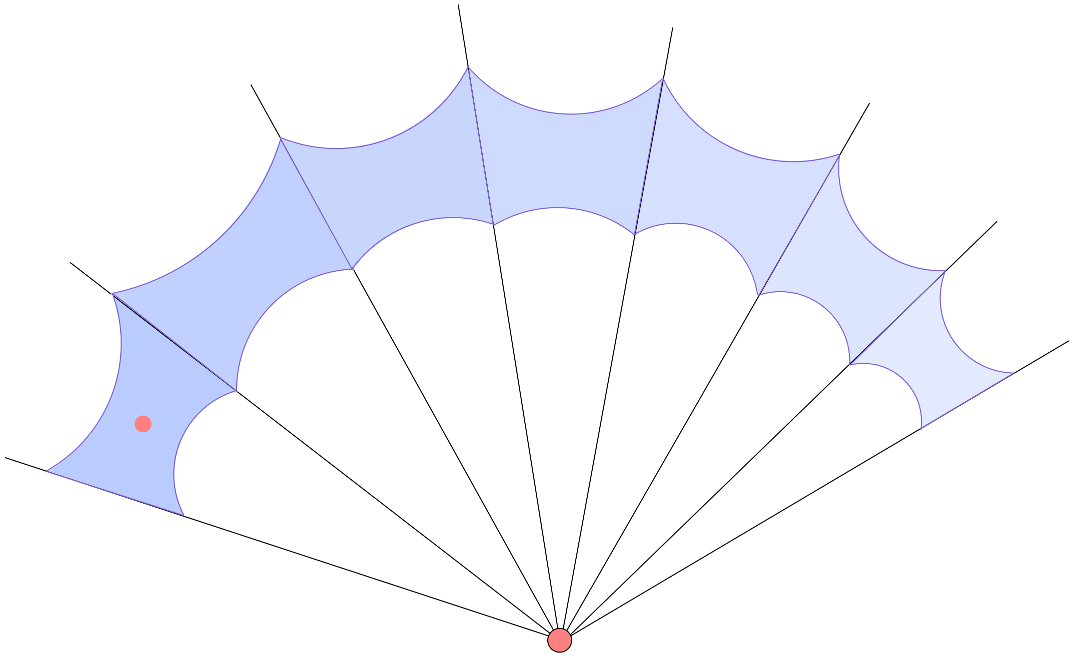

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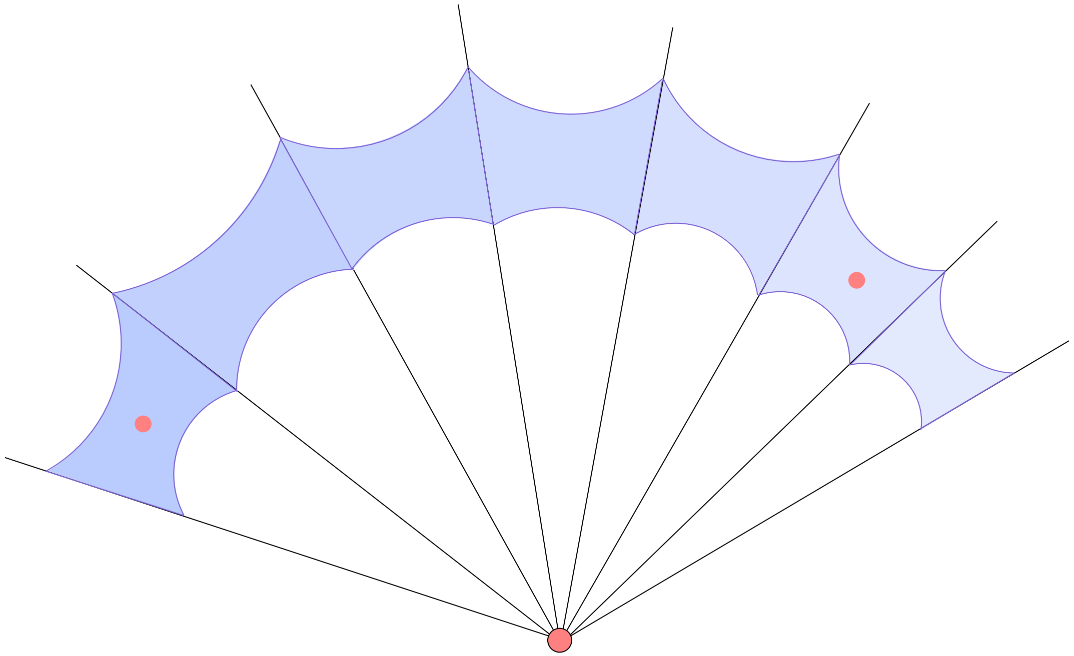

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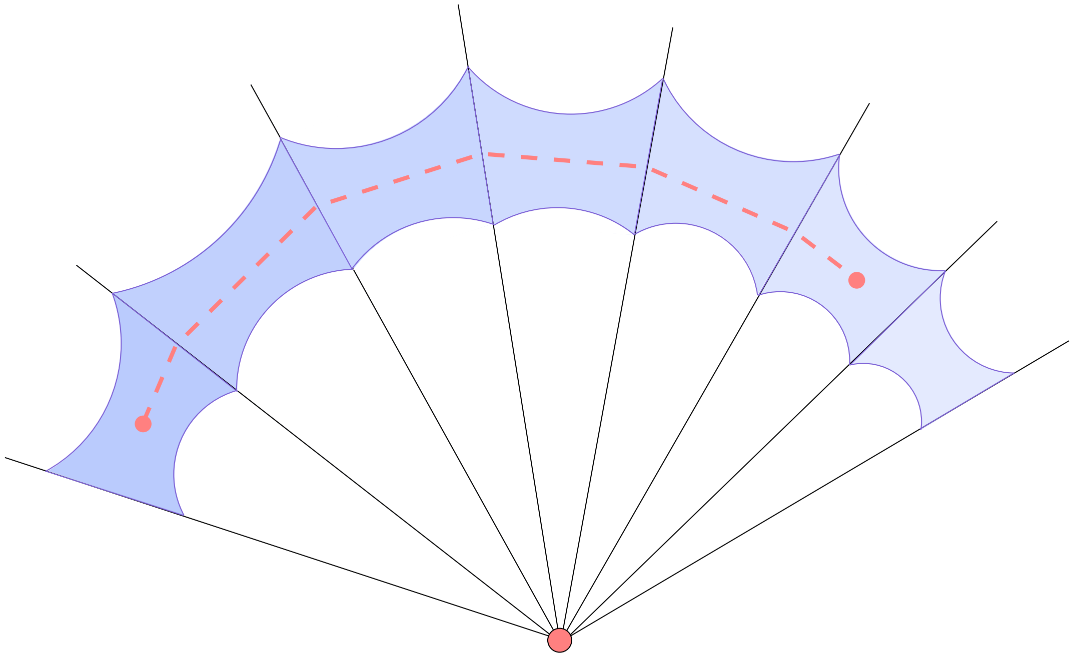

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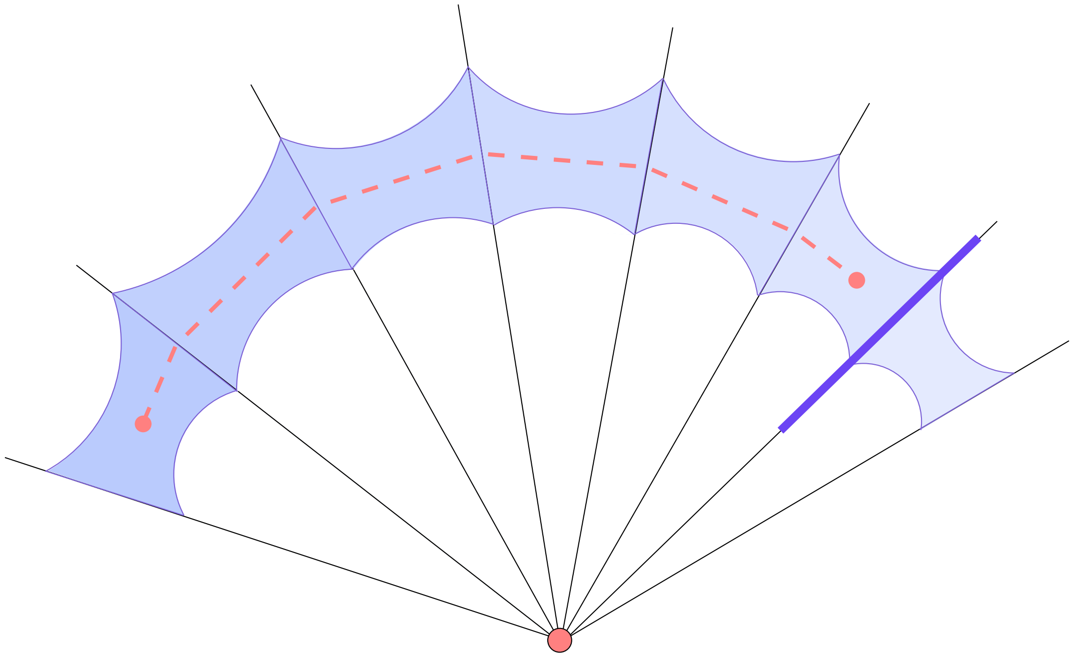

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