Surface Flux Transport

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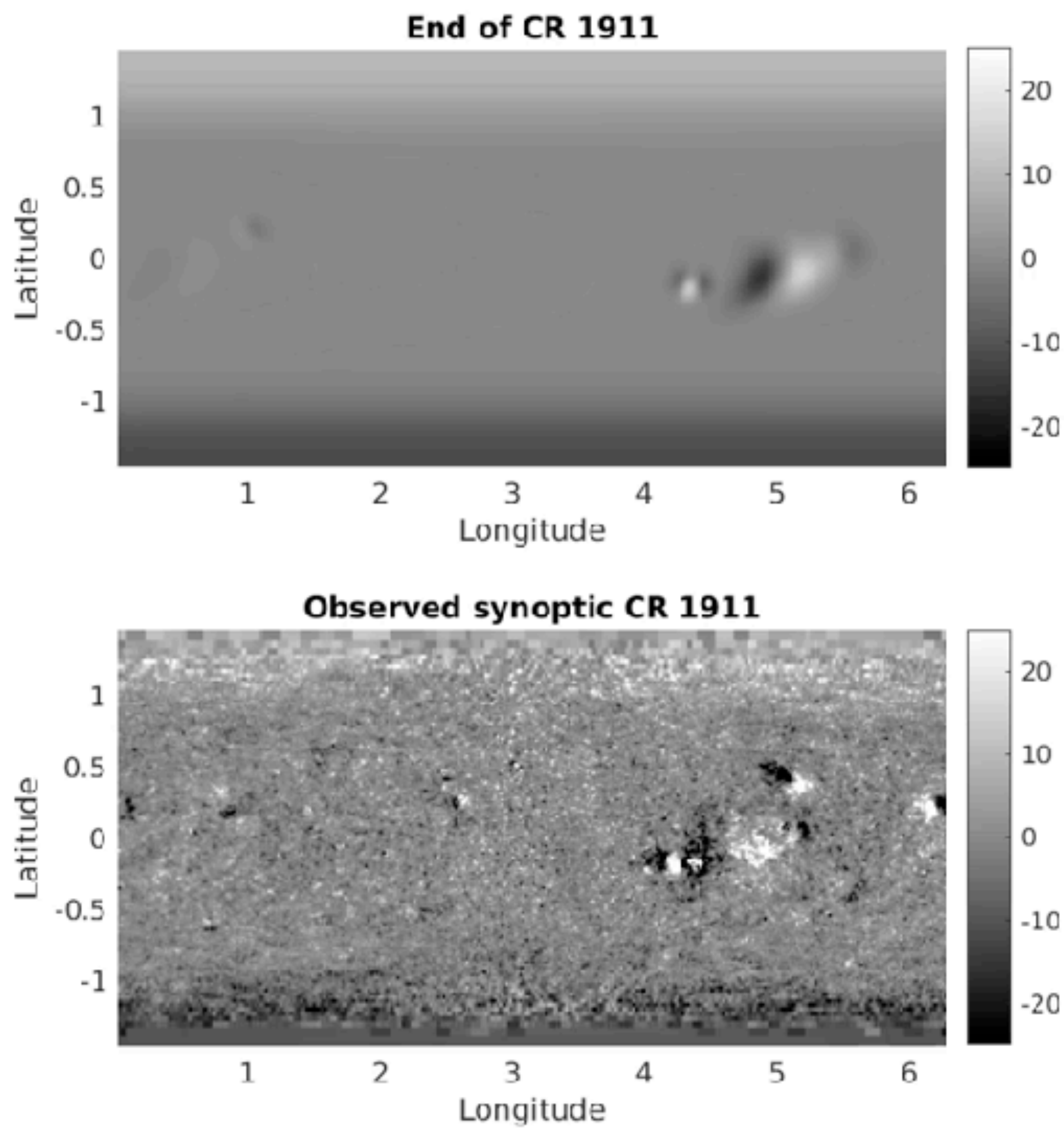
The surface flux transport (SFT) model

Radial magnetic field on the solar surface behaves like a passive scalar.

Leighton [1964]



SFT model

observations [NSO]

The surface flux transport (SFT) model

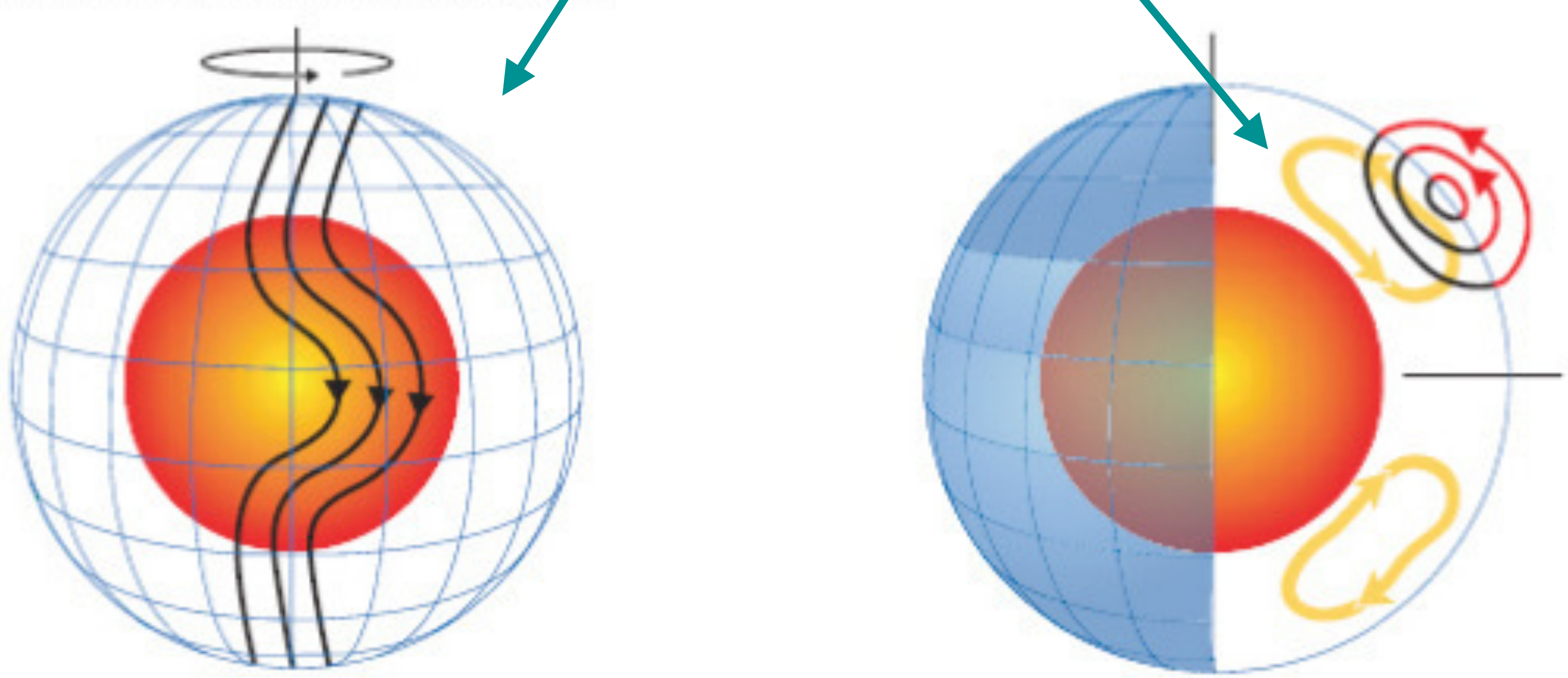
Radial magnetic field on the solar surface behaves like a passive scalar.

$$\frac{\partial B_r}{\partial t} + \nabla_h \cdot (\mathbf{u}_h B_r) = D\nabla_h^2 B_r + S(\theta, \phi, t)$$

Leighton [1964]

- Large-scale flows include **differential rotation** and **meridional flow**:

$$\mathbf{u}_h = R_\odot \sin\theta\,\Omega(\theta)\mathbf{e}_\phi + u_\theta(\theta)\mathbf{e}_\theta$$



adapted from Petrie [2015]

The surface flux transport (SFT) model

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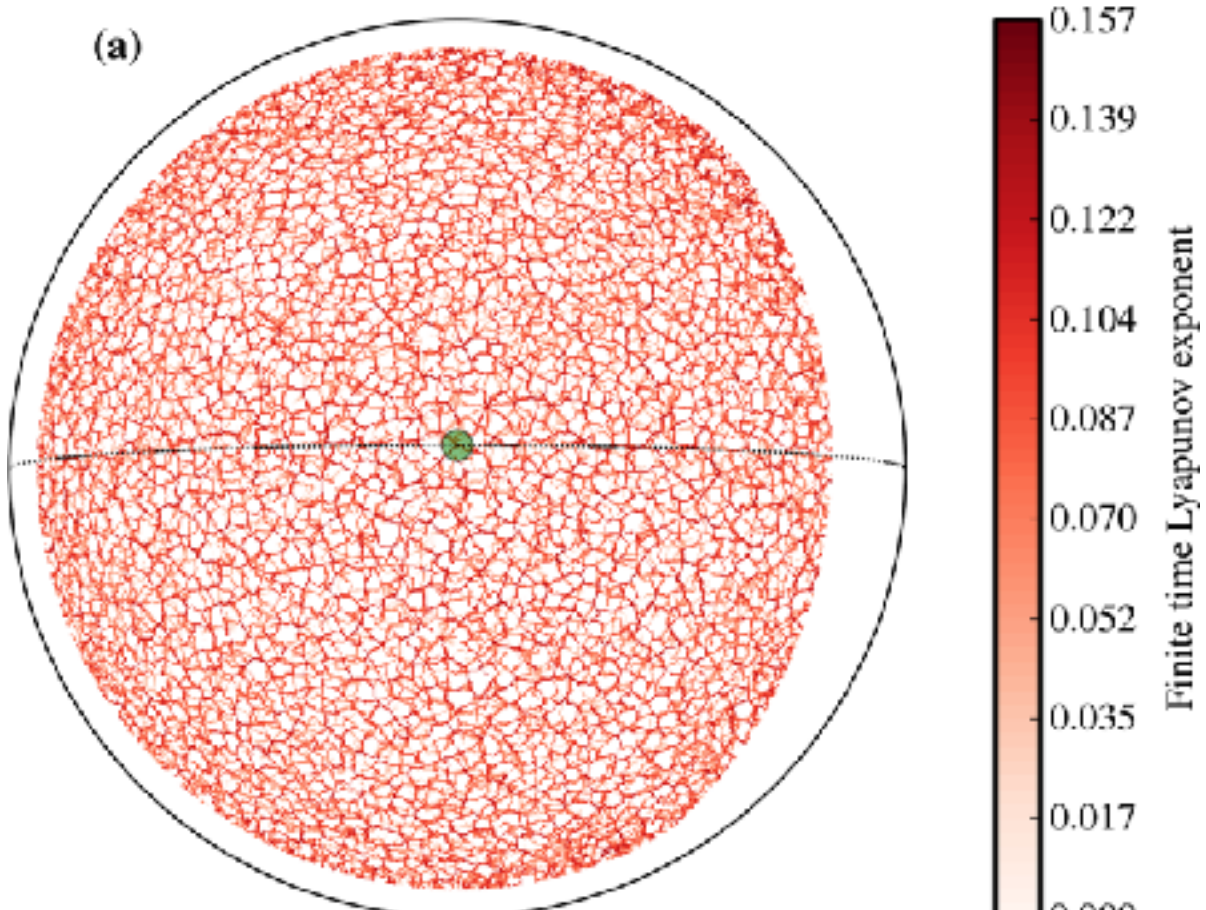
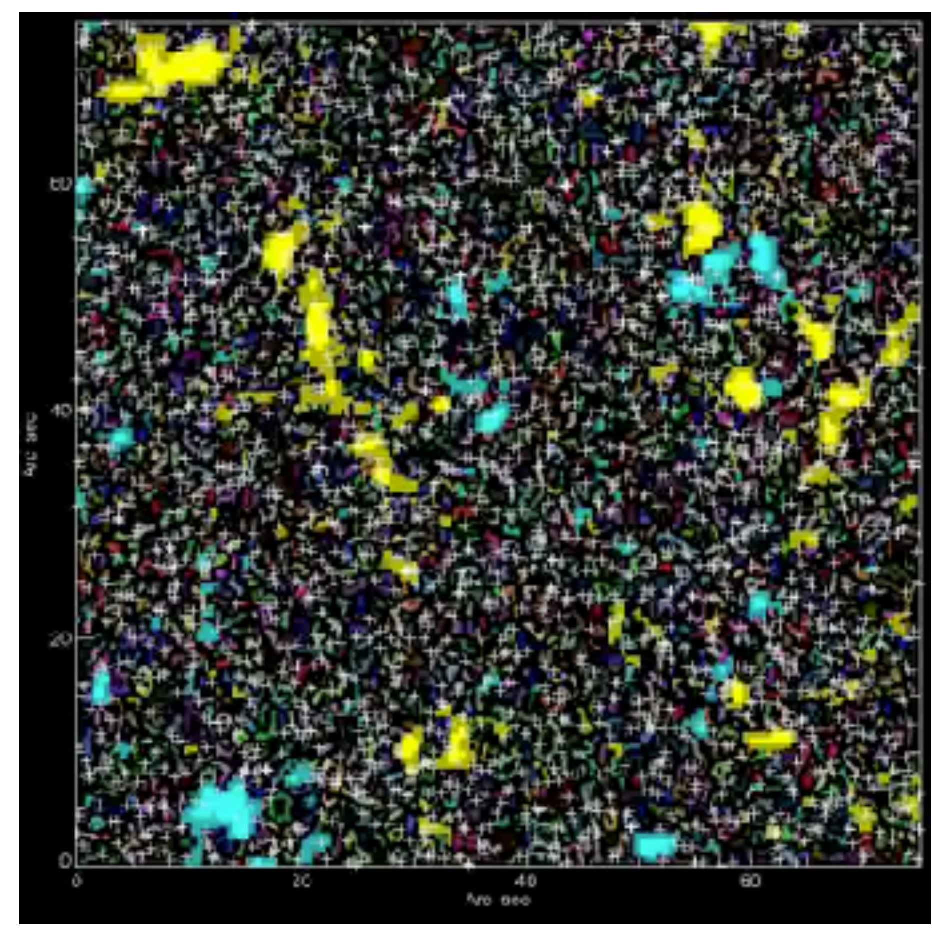
- Large-scale flows include **differential rotation** and **meridional flow.**
- Small-scale convective flows are parametrised by (uniform) **diffusion**:



SDO data showing supergranulation:
Rincon & Rieutord [2018].



Hinode and SDO data (24 hrs):
Roudier *et al.* [2016]

The surface flux transport (SFT) model

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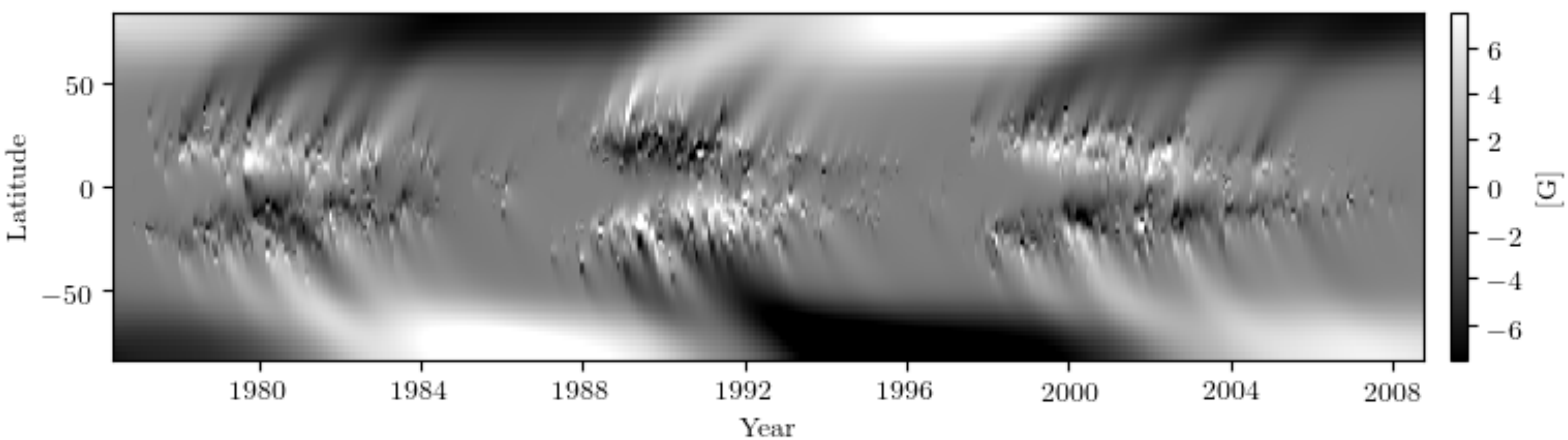
$$\frac{\partial B_r}{\partial t} + \nabla_h \cdot (\mathbf{u}_h B_r) = D\nabla_h^2 B_r + S(\theta, \phi, t)$$

Leighton [1964]

- Large-scale flows include **differential rotation** and **meridional flow**.
- Small-scale convective flows are parametrised by (uniform) **diffusion**.
- The **source term** (emerging active regions) is imposed rather than determined self-consistently.

e.g. "butterfly diagram" showing longitude-averaged field $\langle B_r \rangle(\theta, t) := \frac{1}{2\pi}\int_0^{2\pi} B_r(\theta, \phi, t)\,\mathrm{d}\phi$



Focus: predicting the polar field

- The end-of-cycle polar field is often quantified by the **axial dipole moment**:

$$b_{1,0} := \frac{3}{4\pi}\int_0^{2\pi}\int_0^{\pi} B_r \cos\theta \sin\theta \, \mathrm{d}\theta \mathrm{d}\phi = \frac{3}{2}\int_0^{\pi} \langle B_r\rangle \cos\theta \sin\theta \, \mathrm{d}\theta$$

- For this application it suffices to solve the longitude-averaged equation:

$$\frac{\partial B_r}{\partial t} + \frac{1}{R_\odot \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta u_\theta(\theta) B_r\right) + \Omega(\theta)\frac{\partial B_r}{\partial\phi} = \frac{D}{R_\odot^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial B_r}{\partial\theta}\right) + \frac{D}{R_\odot^2\sin^2\theta}\frac{\partial^2 B_r}{\partial\phi^2} + S$$

$$\Longrightarrow \boxed{\frac{\partial\langle B_r\rangle}{\partial t} + \frac{1}{R_\odot \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta u_\theta(\theta)\langle B_r\rangle\right) = \frac{D}{R_\odot^2 \sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\langle B_r\rangle}{\partial\theta}\right) + \langle S\rangle}$$

- Rest of this talk:

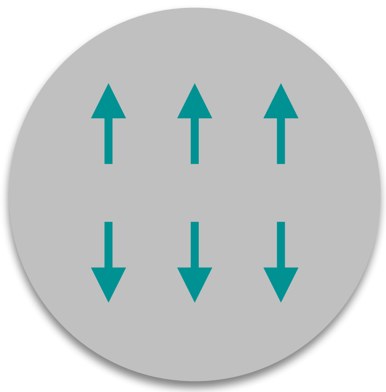
  1. **Flow parameters**
  2. **Source term**
  3. **Physical justification**

1. Flow parameters

The importance of meridional flow

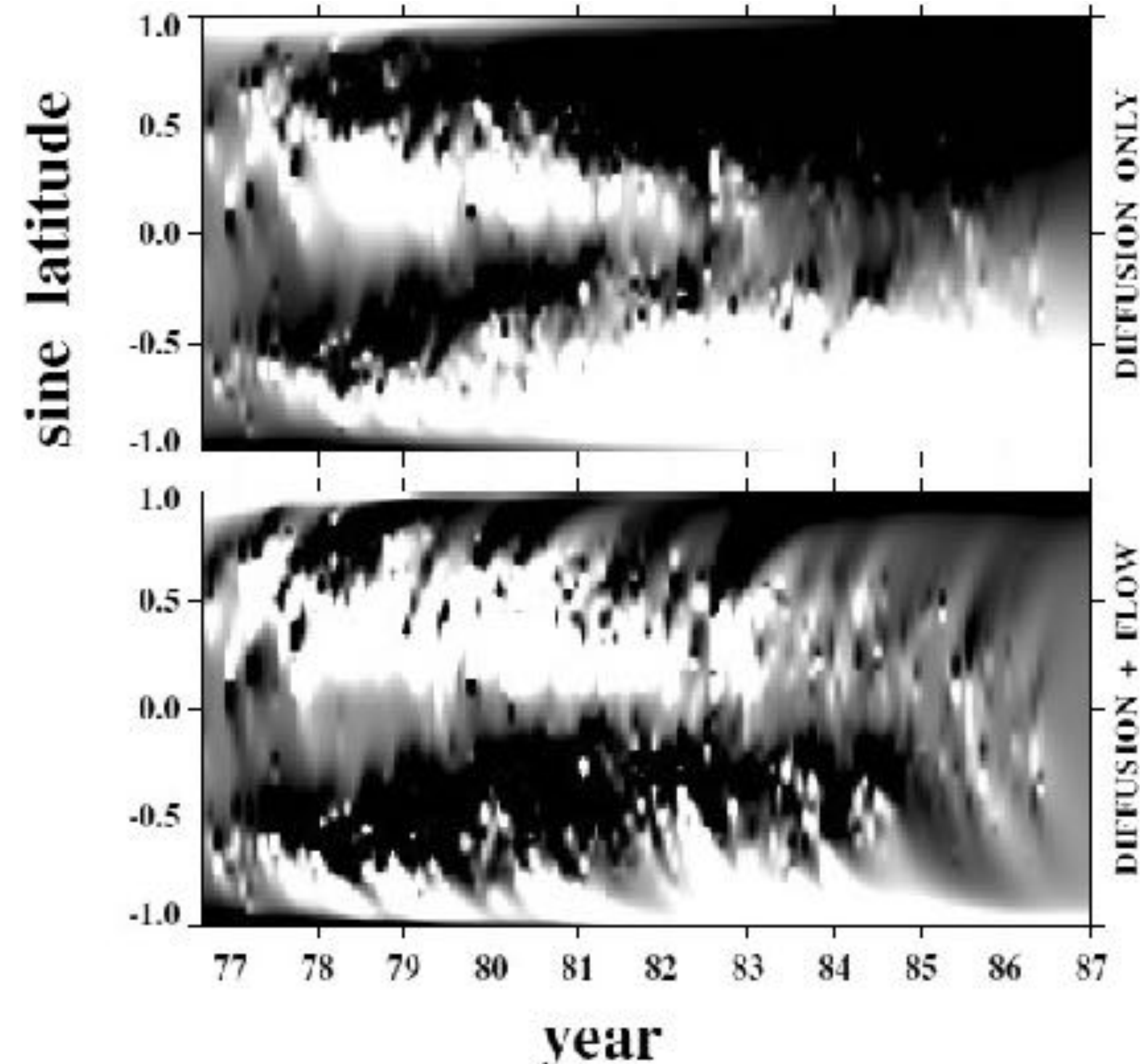
- More realistic large-scale fields are obtained if a large-scale poleward flow is included.

$$\mathbf{u}_h = R_\odot \sin\theta\,\Omega(\theta)\mathbf{e}_\phi + u_\theta(\theta)\mathbf{e}_\theta$$



u_θ

e.g. Sheeley [2005]



- This term permits non-trivial (almost-)steady states:

$$\frac{\partial\langle B_r\rangle}{\partial t} + \frac{1}{R_\odot \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, u_\theta(\theta)\langle B_r\rangle\right) = \frac{D}{R_\odot^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\langle B_r\rangle}{\partial\theta}\right)$$

DeVore, Sheeley & Boris [1984]
van Ballegooijen, Cartledge & Priest [1998]

Axisymmetric steady states

- The latitudinal profile can be computed directly.

e.g. Yeates [2020]

$$u_\theta(\theta) = U\cos\theta\sin^p\theta$$

- Peak flow

$$\max_\theta u_\theta = \pm U p^{p/2}(1+p)^{-(1+p)/2}$$

at

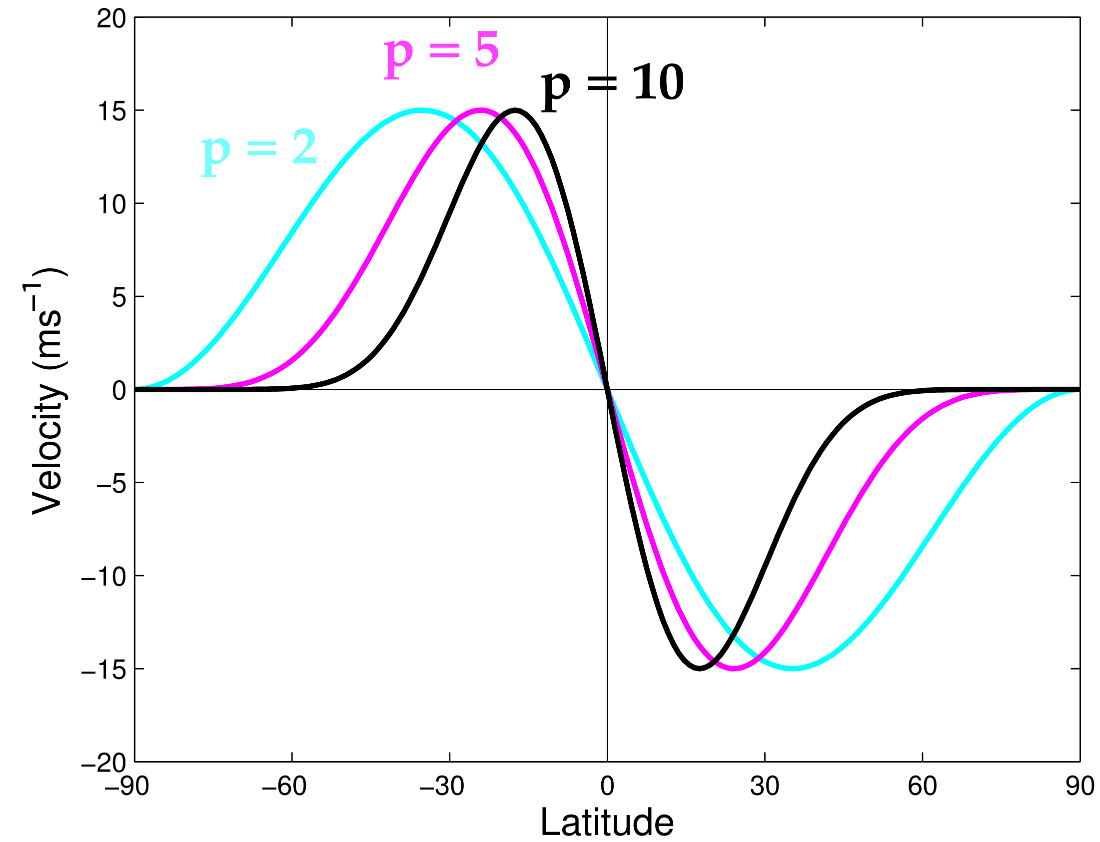
$$\cos\theta = \pm(1+p)^{-1/2}$$

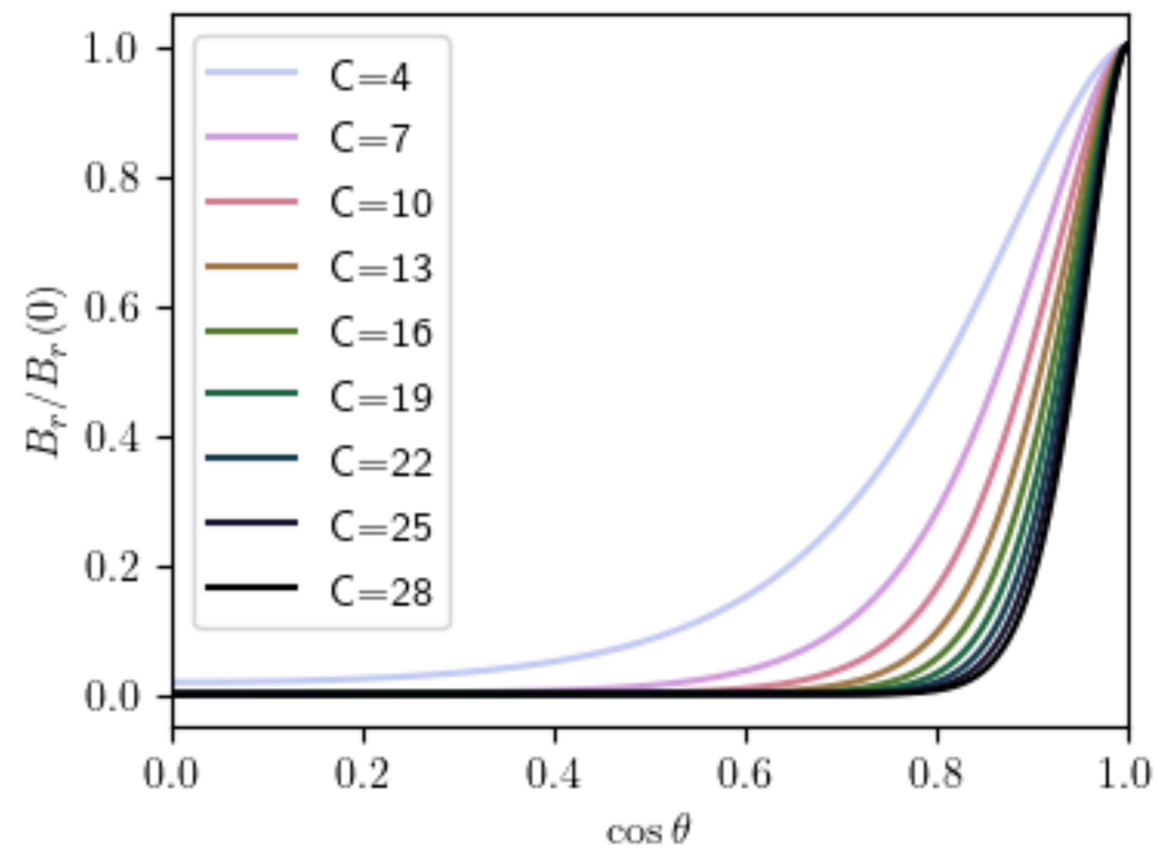
- Almost-steady state solution
  [decays only due to jump at equator]

$$B_r(\theta) = B_r(0)\exp\big(-C\sin^{1+p}\theta\big)$$

where

$$C = \frac{UR_\odot}{(p+1)D}$$





- The amplitude cannot be computed directly as it also depends on the source term.

Parameter optimization

- Recent optimization studies have explored the parameter space of both (meridional) flow speeds and profiles as well as *D* by fitting to observed synoptic data.

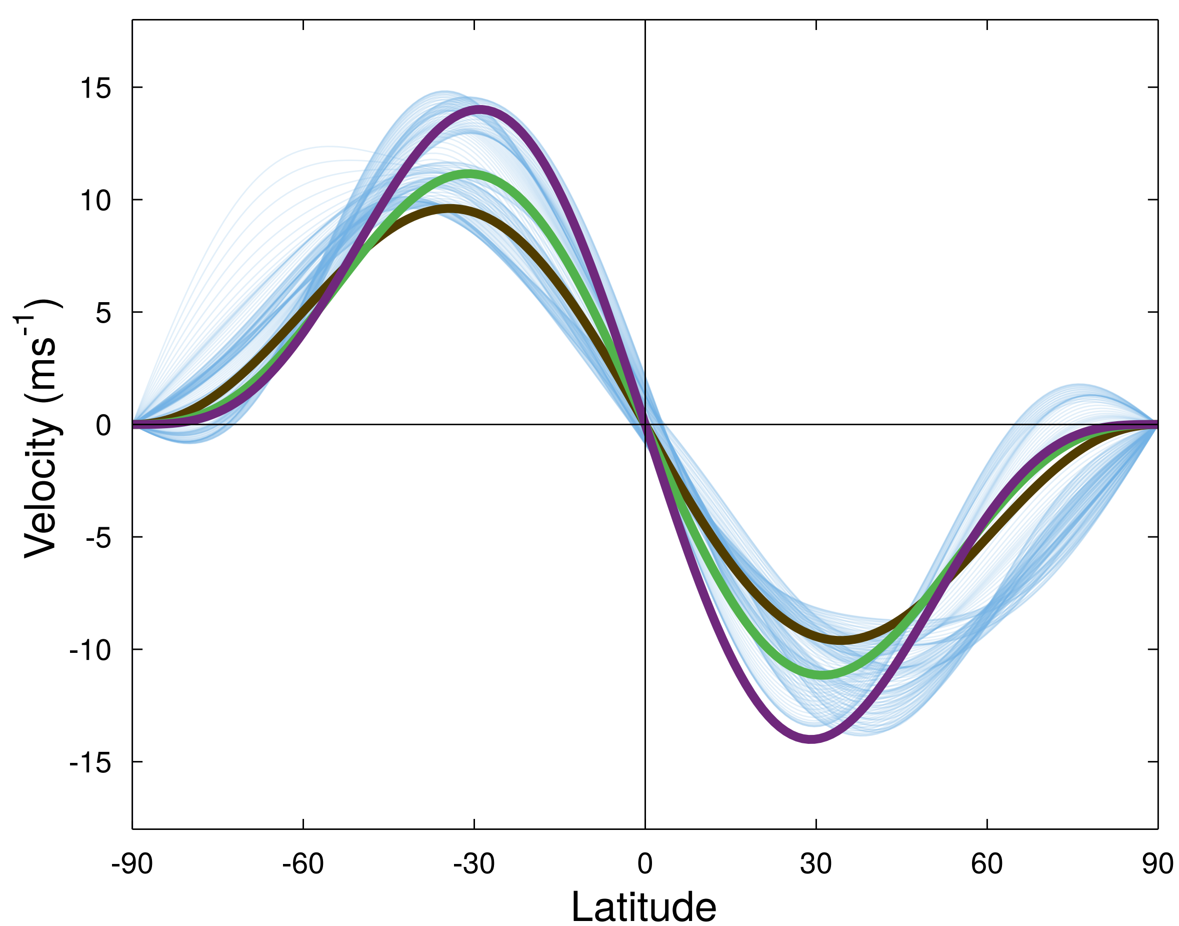
  With BMR sources for Cycle 21: Lemerle, Charbonneau & Carignan-Dugas [2015]

  With observed sources for Cycles 21-24: Whitbread *et al.* [2017]

  With synthetic source (1d model): Petrovay & Talafha [2019]

- Optimal flow profiles are consistent with observations.

e.g. Whitbread *et al.* [2017]



optimum model

optimum BMR model

optimum model with decay

observed flow profiles from magnetic feature tracking (D. Hathaway)

Parameter optimization

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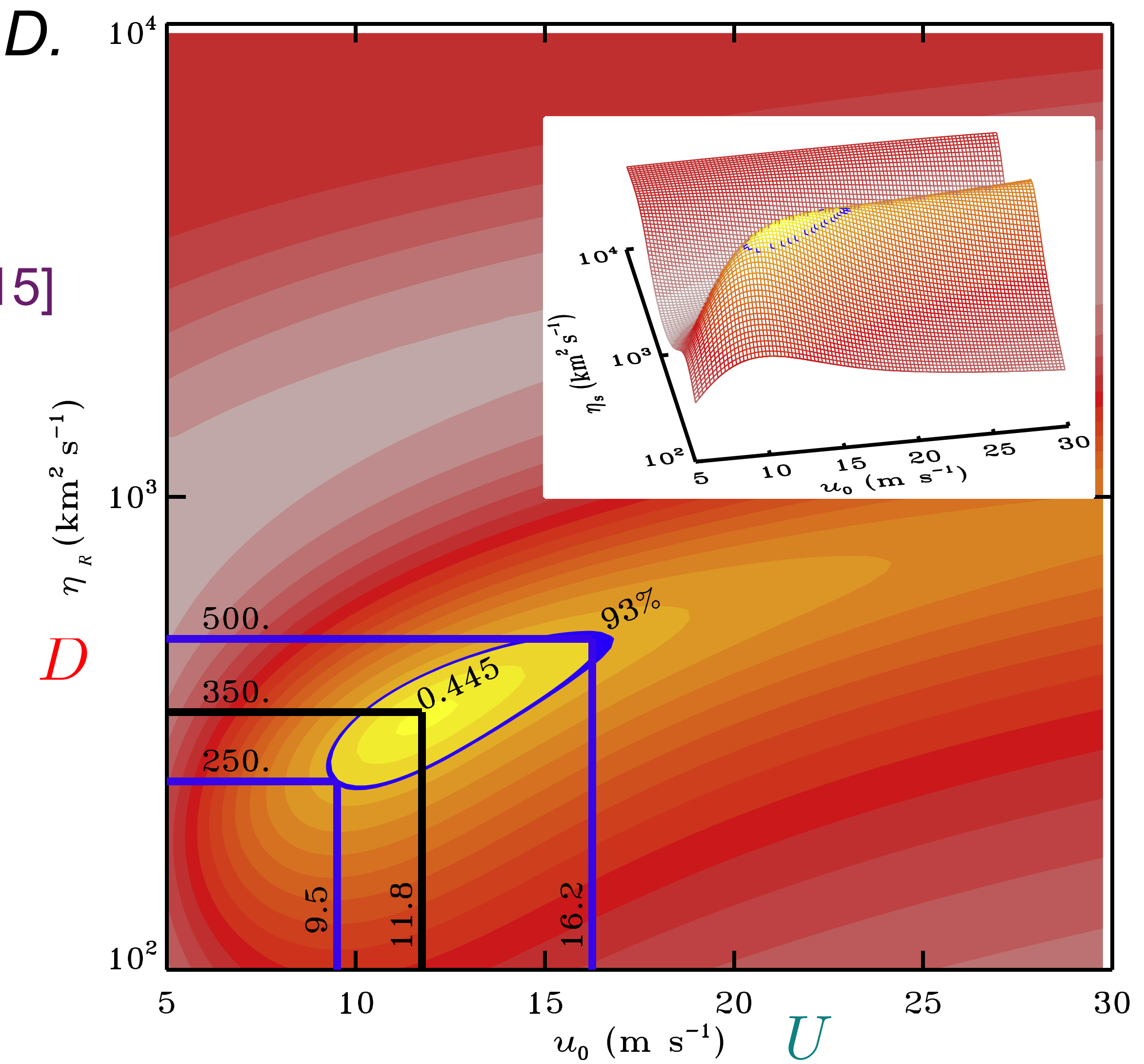
  With synthetic source (1d model): Petrovay & Talafha [2019]

- Optimal flow profiles are consistent with observations.
- There is a degeneracy between flow speed and *D*.

  e.g. Lemerle, Charbonneau & Carignan-Dugas [2015]



- Fit can be further improved by varying the flow speed between solar cycles (or even within a cycle).

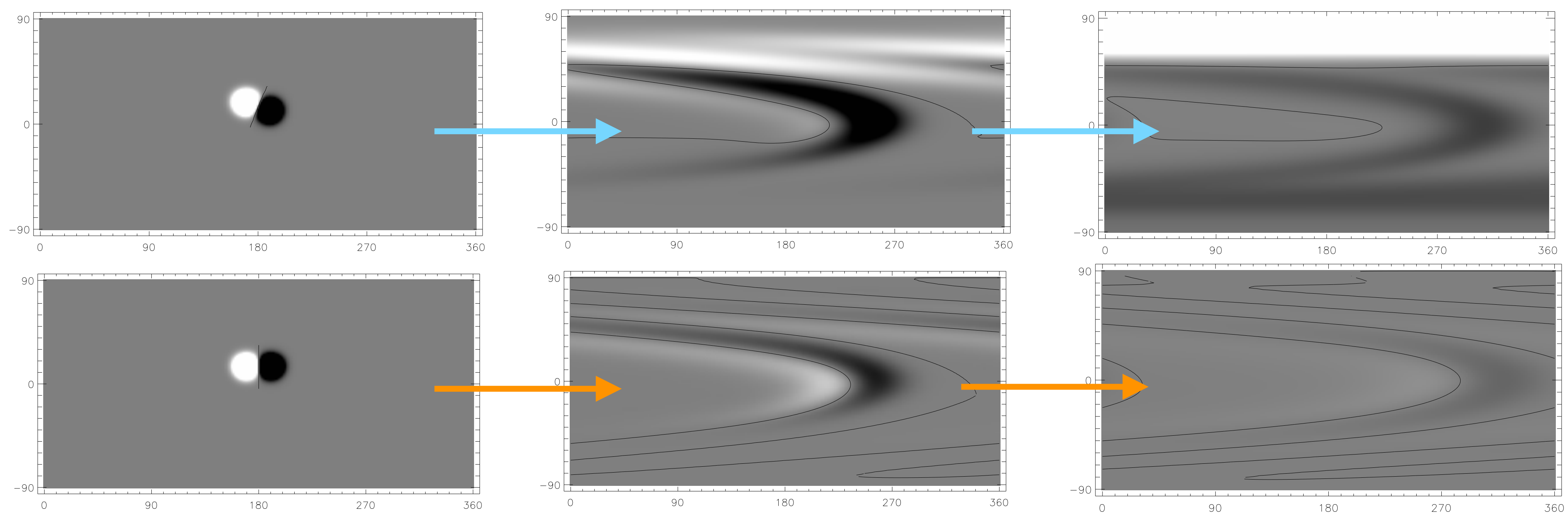
2. Source term

Contribution of an individual bipolar magnetic region (BMR)

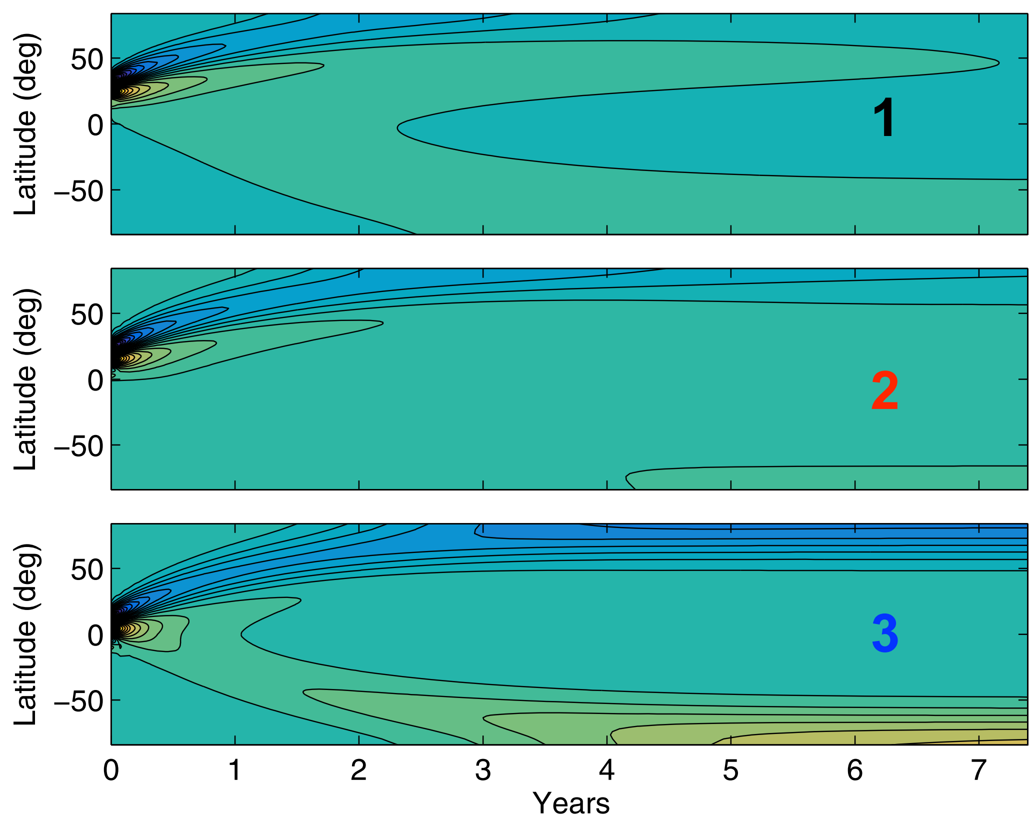
- Since the SFT equation (1D or 2D) is **linear**, the polar field is the sum of solutions for each individual active region source.

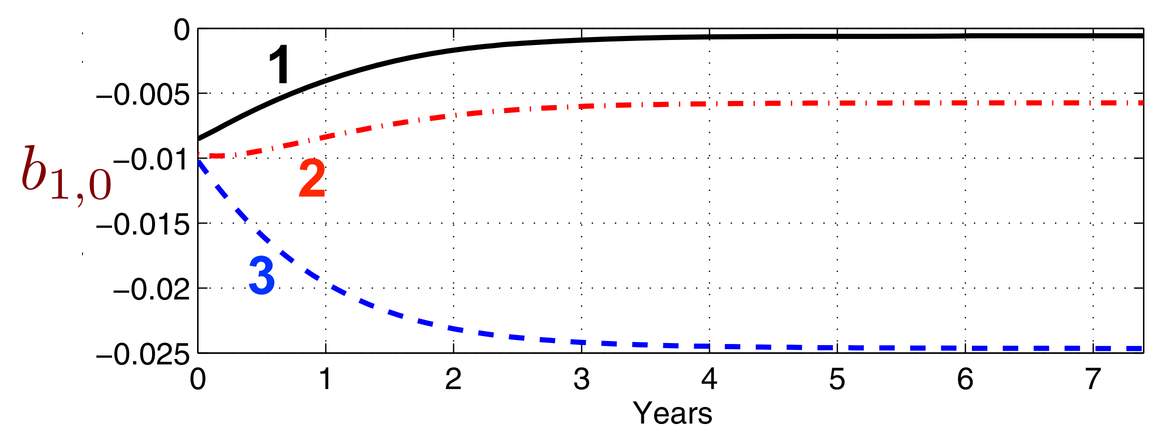
e.g. Mackay & Yeates [2012]

after 15 rotations **after 30 rotations**



e.g. Yeates, Baker & van Driel-Gesztelyi [2015]





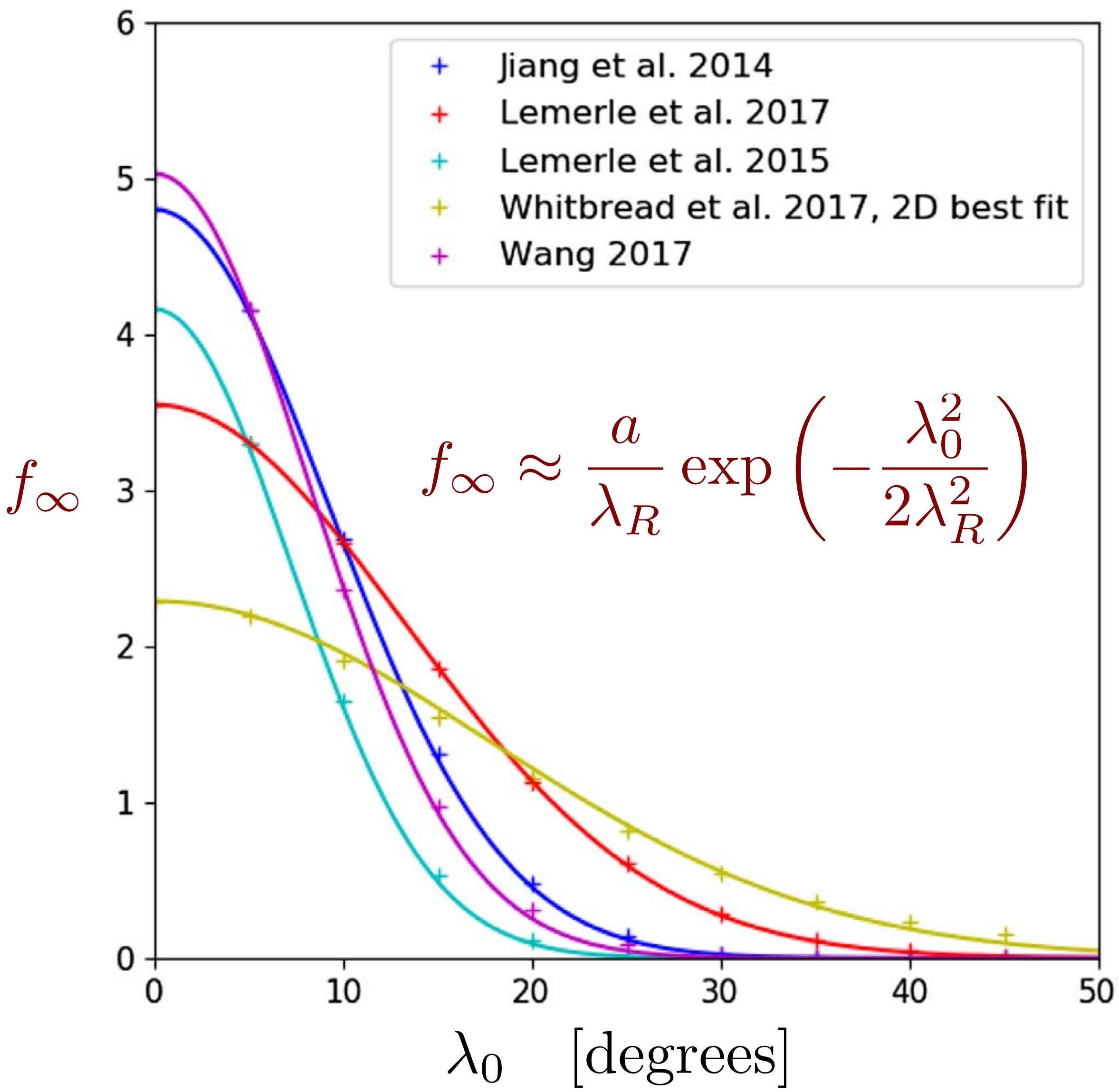
- Active regions which are more tilted or closer to the equator contribute more to the polar field.

cf. "rogue regions":
Cameron et al. [2013]; Nagy et al. [2017]

Dipole amplification factor – ratio $f_\infty := \dfrac{b_{1,0}(t \to \infty)}{b_{1,0}(t = 0)}$

- For a **BMR** (**bipolar magnetic region**) source, the dipole “amplification” is approximately a Gaussian function of emergence latitude.

Jiang, Cameron & Schüssler [2014]



Mathematical justification:
Petrovay, Nagy & Yeates [2020]

- Width depends on the transport parameters:

$$\lambda_R = \sqrt{\sigma_0^2 + \frac{D}{UR_\odot}}$$

- Derivation works by noticing that:
  1. Net flux across equator is key;
  2. Near equator we can use a Cartesian approximation where SFT equation can be integrated exactly.

Generalised to non-BMRs by Wang, Jiang & Wang [2021]

- This determines which active regions are important for the global dipole.

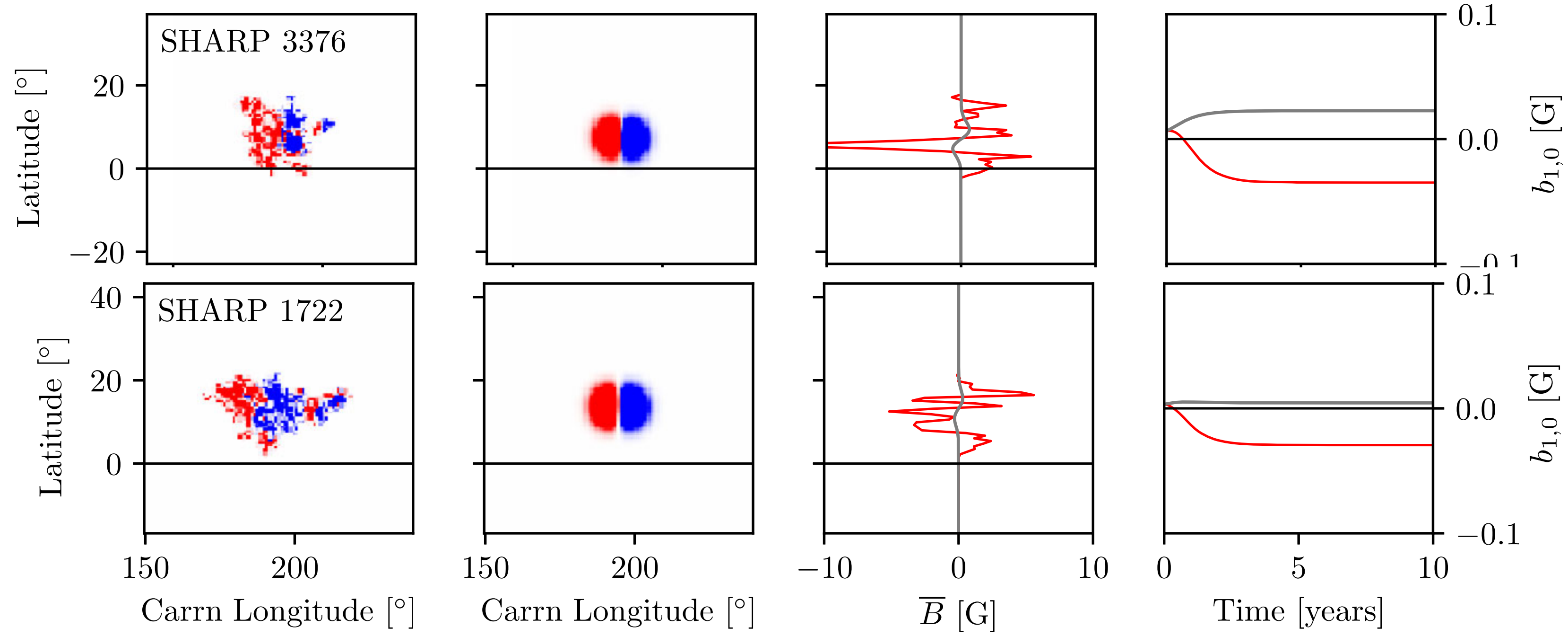
Concept of “ARDoR” (active region degree of rogueness) – Petrovay et al. [2020]

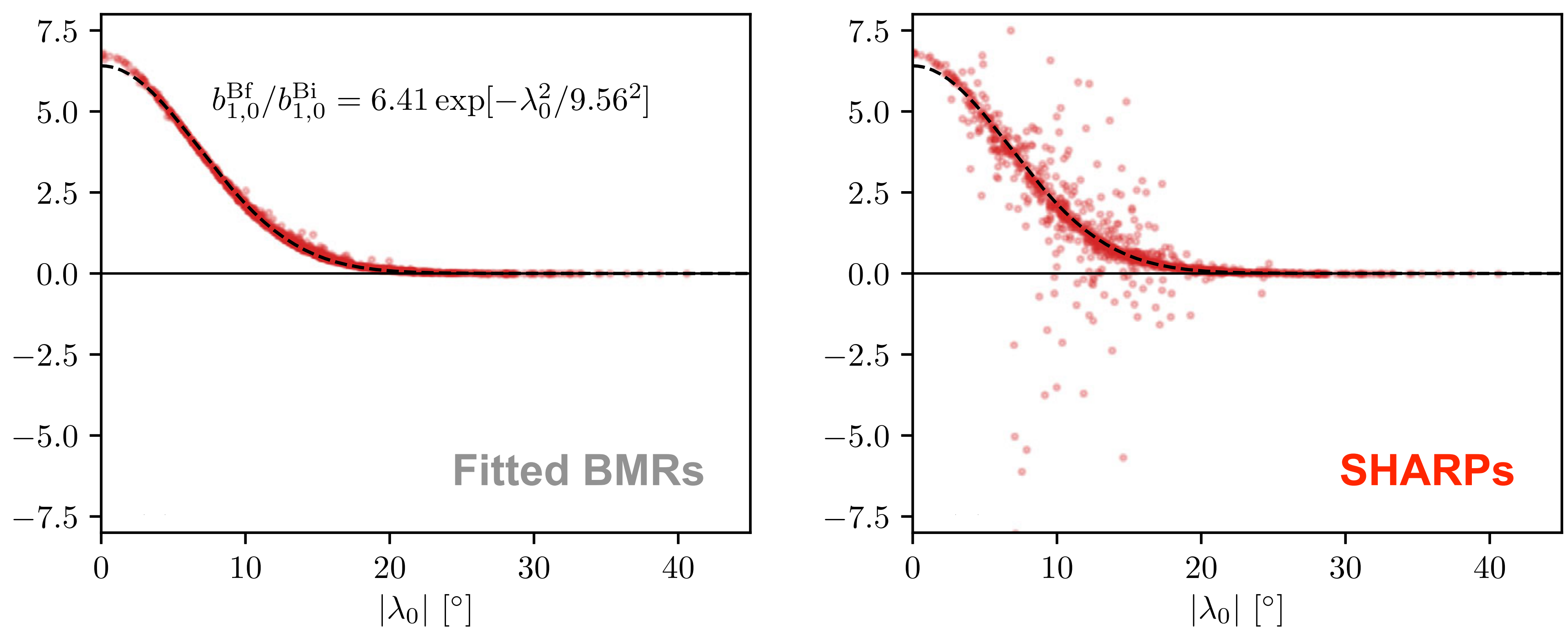
Effect of non-bipolarity

- In reality, irregular-shaped regions may have different dipole amplification.

  e.g. asymmetric polarities Iijima, Hotta & Imada [2019]; **delta-spot regions** Jiang et al. [2019]

Study comparing HMI/SHARPs with fitted BMRs Yeates [2020]





- Fitting BMRs with matching dipoles leads to 24% overestimate of end-of-cycle dipole.

3. Physical justification

Derivation from MHD induction equation

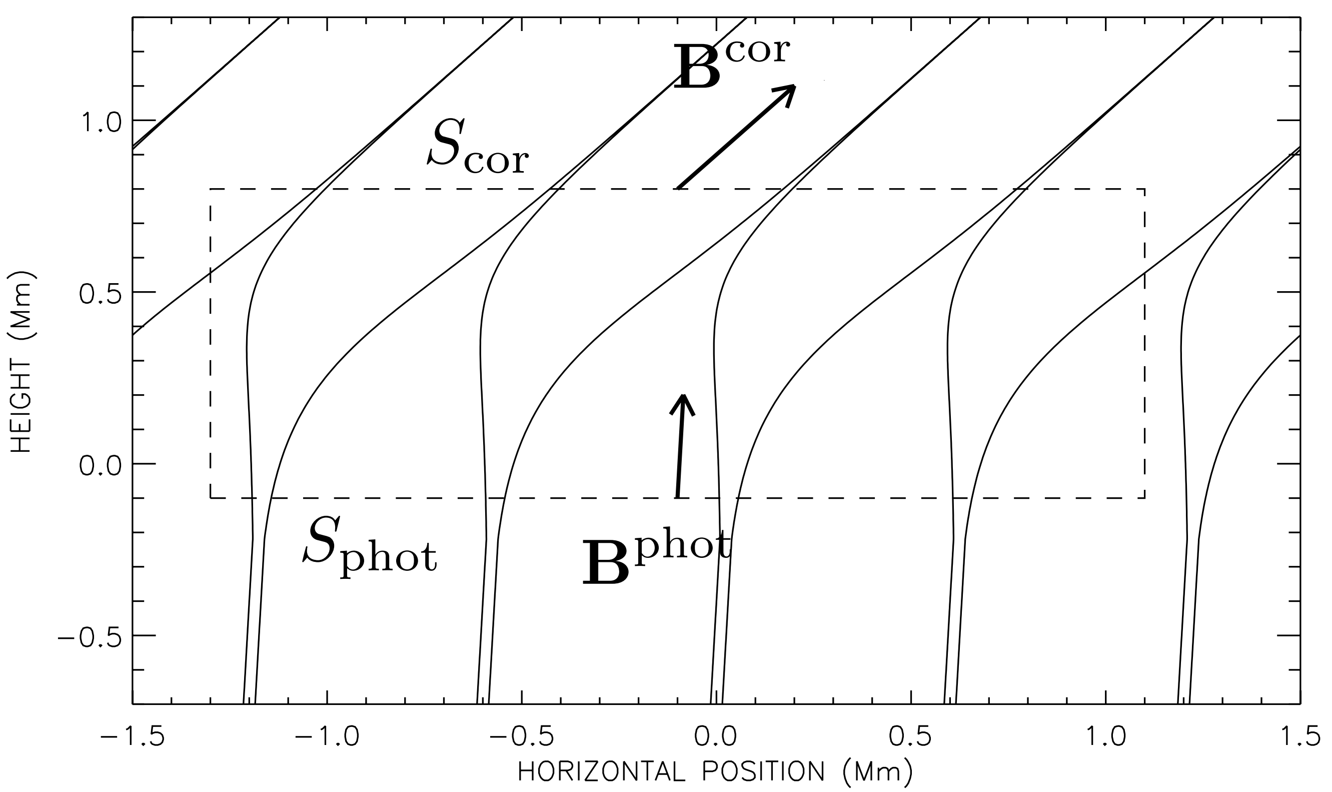
turbulent diffusivity

$$\frac{\partial B_r}{\partial t} = \mathbf{e}_r \cdot \nabla \times \left(\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}\right)$$

1. Assuming $u_r = 0$, first term gives $\mathbf{e}_r \cdot \nabla \times \left(\mathbf{u} \times \mathbf{B}\right) = -\nabla_h \cdot \left(\mathbf{u}_h B_r\right)$

2. Simple conservation laws imply photospheric **B** is approximately radial:

van Ballegooijen & Mackay [2007]



(i) Assume photospheric field concentrated in kG flux tubes:

$$\mathbf{B}^{\rm phot} = f\mathbf{B}^{\rm tube} \qquad f \ll 1$$

(ii) Conservation of flux through $S_{\rm phot}, S_{\rm cor}$

$$\Longrightarrow \quad B_r^{\rm cor} = B_r^{\rm phot}$$

(iii) Force balance

$$\int_{S_{\rm cor}} \boldsymbol{\sigma} \cdot \mathbf{e}_r \,\mathrm{d}S - \int_{S_{\rm phot}} \boldsymbol{\sigma} \cdot \mathbf{e}_r \,\mathrm{d}S + m\mathbf{g} = \mathbf{0}$$

$$\boldsymbol{\sigma} = -\left(p + \frac{B^2}{8\pi}\right)\boldsymbol{I} + \frac{\mathbf{BB}}{8\pi} \quad \Longrightarrow \quad \begin{array}{l} B_\theta^{\rm cor} B_r^{\rm cor} - f B_\theta^{\rm tube} B_r^{\rm tube} = 0 \\ B_\phi^{\rm cor} B_r^{\rm cor} - f B_\phi^{\rm tube} B_r^{\rm tube} = 0 \end{array}$$

Solving simultaneously gives $B_\theta^{\rm phot} = f B_\theta^{\rm cor} \qquad B_\phi^{\rm phot} = f B_\phi^{\rm cor}$

Derivation from MHD induction equation

turbulent diffusivity

$$\frac{\partial B_r}{\partial t} = \mathbf{e}_r \cdot \nabla \times \Big(\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B} \Big)$$

1. Assuming $u_r = 0$, first term gives $\mathbf{e}_r \cdot \nabla \times \Big(\mathbf{u} \times \mathbf{B} \Big) = -\nabla_h \cdot \Big(\mathbf{u}_h B_r \Big)$
2. Simple conservation laws imply photospheric **B** is approximately radial.
3. The diffusion term leads to an additional radial gradient term:

Assuming $\eta = \eta(r)$ gives

$$-\mathbf{e}_r \cdot \nabla \times \Big(\eta \nabla \times \mathbf{B} \Big) = \frac{\eta}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial B_r}{\partial \theta} \right) + \frac{\eta}{r^2 \sin^2\theta} \frac{\partial^2 B_r}{\partial \phi^2} - \frac{\eta}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta B_\theta + r \sin\theta \frac{\partial B_\theta}{\partial r} \right) - \frac{\eta}{r^2 \sin\theta} \frac{\partial}{\partial \phi} \left(B_\phi + r \frac{\partial B_\phi}{\partial r} \right)$$

Assuming $B_\theta = B_\phi = 0$ on the photosphere, and using $\nabla \cdot \mathbf{B} = 0$, gives

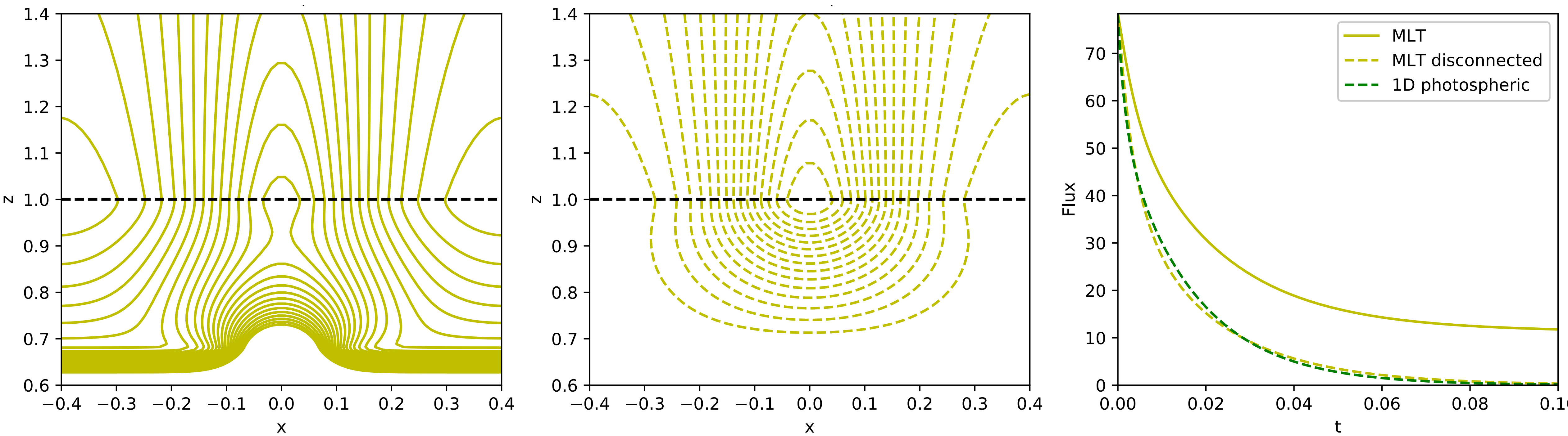
$$-\mathbf{e}_r \cdot \nabla \times \Big(\eta \nabla \times \mathbf{B} \Big) = \frac{\eta}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial B_r}{\partial \theta} \right) + \frac{\eta}{r^2 \sin^2\theta} \frac{\partial^2 B_r}{\partial \phi^2} + \frac{\eta}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} \big(r^2 B_r \big) \right)$$

Additional "radial diffusion" term missing from SFT

Effect of the missing radial diffusion?

$$+\frac{\eta}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r^2 B_r)\right)$$

- This term can have either sign, depending on $\eta(r)$ and on the shape of the subsurface magnetic field.
  - e.g. if active regions remain connected to the base of the convection zone, this **slows** diffusion of B_r at the surface: Whitbread, Yeates & Muñoz-Jaramillo [2019]



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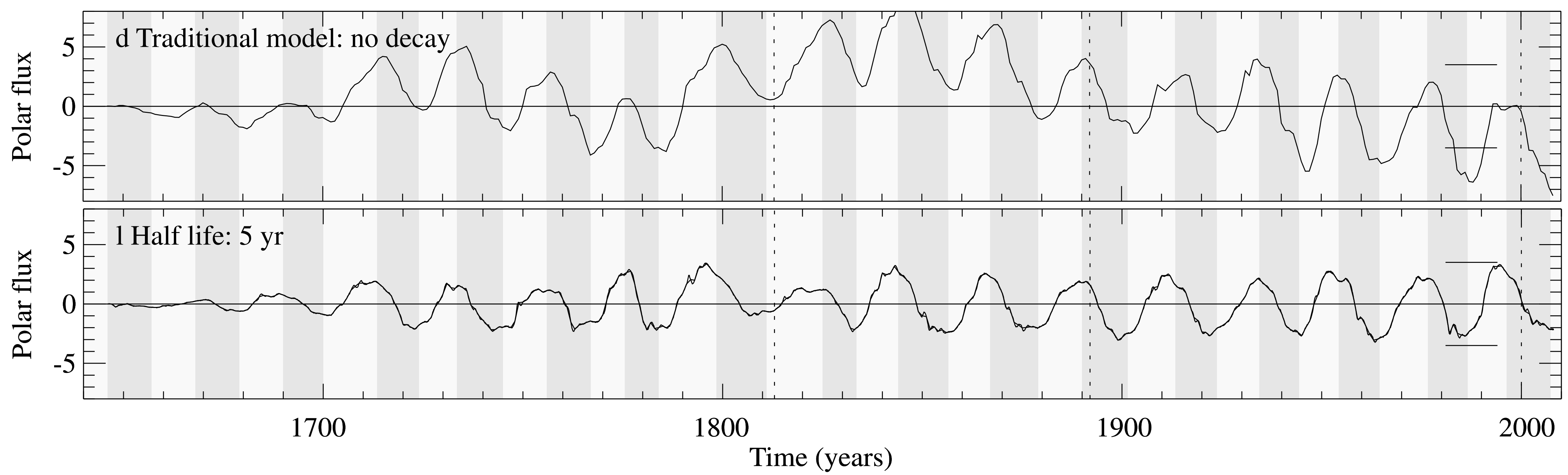
- The crudest model is a uniform exponential decay. $-\frac{B_r}{\tau}$

Proposed to fix long-term drift of polar field: Schrijver, DeRosa & Title [2002]



For a single cycle, whether it is needed depends on the model:
Petrovay & Talafha [2019] vs. Whitbread *et al.* [2017] or Yeates [2020]

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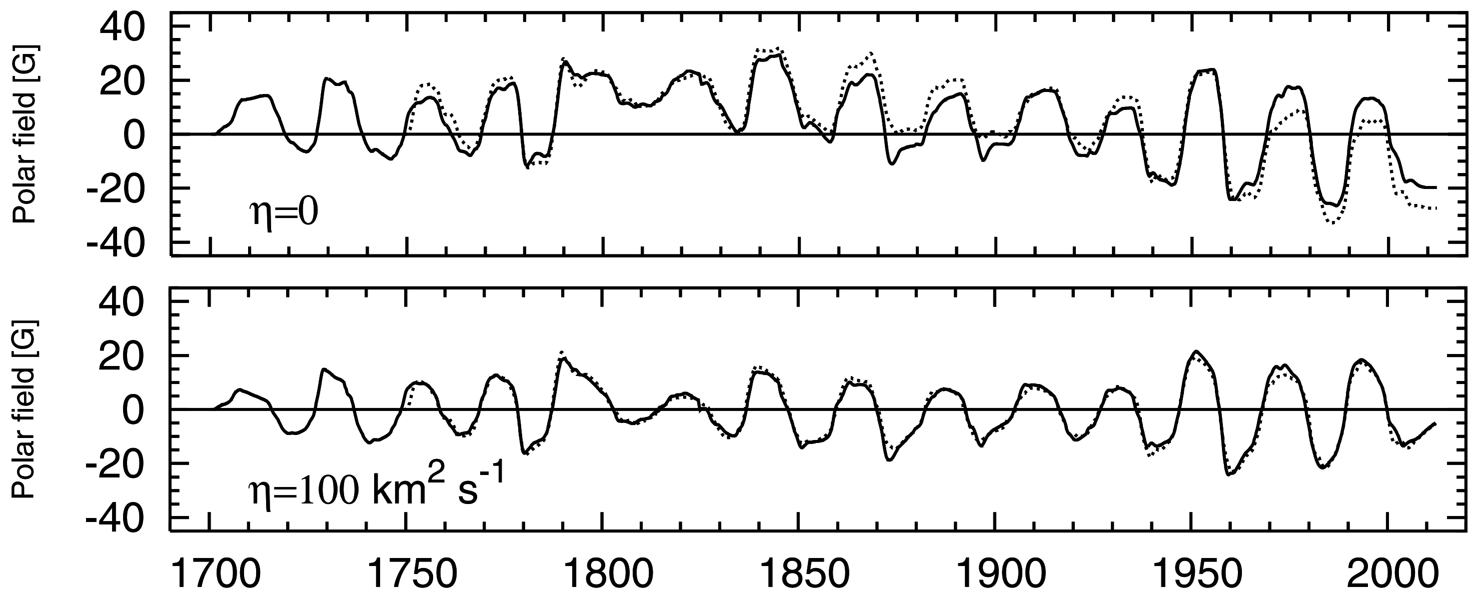
- This term can have either sign, depending on $\eta(r)$ and on the shape of the subsurface magnetic field.
  - e.g. if active regions remain connected to the base of the convection zone, this slows diffusion of B_r at the surface.
- The crudest model is a uniform exponential decay. $-\frac{B_r}{\tau}$
- An improved model is to assume uniform diffusion (no flow) in the convection zone.

Baumann, Schmitt & Schüssler [2006]

Spherical harmonic modes with higher wavenumber decay faster.

For $\eta = 100\,\mathrm{km}^2\mathrm{s}^{-1}$ the largest scale mode decays in about 5.2 years.



Outlook

- The SFT model is one of the success stories of 20^{th} Century Solar Physics!
- Some outstanding issues:
  - More accurate models of convection?
    cf. Worden & Harvey [2000], Upton & Hathaway [2014]
  - Importance/form of nonlinearities in flows/diffusion?
    cf. DeRosa & Schrijver [2006], Cameron & Schüssler [2010]
  - Improving historical models?
    cf. Schrijver, DeRosa & Title [2002], Jiang *et al*. [2011], Virtanen *et al.* [2017]
  - Improving data assimilation (e.g. far side)?
    cf. AFT model – Upton & Hathaway [2014]; **ADAPT model** – Hickman *et al.* [2015]
  - Better coupling to dynamo models to "complete the loop"?
    e.g. Miesch & Dikpati [2014], Lemerle & Charbonneau [2017]

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