



North-East and Midlands Stochastic Analysis Seminar supported by the *London Mathematical Society* and *Isaac Newton Institute for Mathematical Sciences*

Organisers:

*Zdzislaw Brzezniak (York), Horatio Boedihardjo (Warwick) David Elworthy (Warwick),
Chunrong Feng (Durham), Maximiliano Gubinelli (Oxford), Zhongmin Qian (Oxford),
Roger Tribe (Warwick), Huaizhong Zhao (Durham)*

Department of Mathematical Sciences, Durham University
Venue: MCS001, 5 July 2023

Talks

14:00-15:00: **Iain Smears (University of College London)**

Numerical methods for Hamilton-Jacobi-Bellman equations

15:00-15:30: Break

15:30-16:30: **Emanuela Gussetti (Bielefeld University)**

An application of rough paths theory to the study of the stochastic Landau-Lifschitz-Gilbert equation

16:30-17:30: **Denis Talay (Ecole Polytechnique)**

Convergence to diffusion invariant measures: a survey of old techniques and related open questions

If you have any queries, please contact Chunrong Feng (chunrong.feng@durham.ac.uk) or Huaizhong Zhao (huaizhong.zhao@durham.ac.uk).



LONDON
MATHEMATICAL
SOCIETY
EST. 1865



Isaac Newton Institute
for Mathematical Sciences



Engineering and
Physical Sciences
Research Council

Emanuela Gussetti, Bielefeld University

Title: An application of rough paths theory to the study of the stochastic Landau-Lifschitz-Gilbert equation

Abstract: The Landau-Lifschitz-Gilbert equation is a model describing the magnetisation of a ferromagnetic material. The stochastic model is studied to observe the role of thermal fluctuations. We interpret the linear multiplicative noise appearing by means of rough paths theory and we study existence and uniqueness of the solution to the equation on a one dimensional bounded domain D . As a consequence of the rough paths formulation, the map that to the noise associates the unique solution to the equation is locally Lipschitz continuous in the strong norm $L^\infty([0,T];H^1(D)) \cap L^2([0,T];H^2(D))$. This implies a Wong-Zakai convergence result, a large deviation principle, a support theorem. We prove also continuity with respect to the initial datum in $H^1(D)$, which allows to conclude the Feller property for the associated semigroup. We also discuss briefly a path-wise central limit theorem and a moderate deviations principle for the stochastic LLG: in this case the rough paths formulation leads to a path-wise convergence, not easily reachable in the classical Itô's calculus setting.

The talk is based on a joint work with A. Hocquet [<https://arxiv.org/abs/2103.00926>] and on [<https://arxiv.org/abs/2208.02136>].

Iain Smears, University of College London

Title: Numerical methods for Hamilton-Jacobi-Bellman equations

Abstract: Hamilton-Jacobi-Bellman (HJB) equations are a broad class of fully nonlinear partial differential equations of second order, which arise in applications of stochastic optimal control. HJB equations broadly include as special cases many other PDE, from linear elliptic/parabolic advection-diffusion-reaction equations to other famous examples of fully nonlinear PDE such as the Monge-Ampere equation. The full nonlinearity of the equations poses special challenges for numerical analysis, especially on essential questions on the well-posedness of numerical discretizations, their stability and their convergence to the exact solution.

The aim of this talk is to give an overview of the state of the art in the numerical analysis of these problems, and we will consider various approaches and their applicability depending on the structure of the underlying problem. We will first discuss the convergence to the viscosity solution of monotone numerical methods, i.e. methods that are designed to satisfy a discrete maximum principle. Under suitable assumptions, convergence of the approximations can be proved even for degenerate elliptic/parabolic problems. Some examples of monotone methods that we consider include upwind finite difference methods and stabilized finite element methods.

However, the construction of methods with discrete maximum principles is not always straightforward and there are some downsides in terms of computational flexibility and

efficiency. Therefore, in the second part of the talk, we will discuss how discrete maximum principles are not always necessary for rigorous numerical analysis. In particular, for HJB equations where the coefficients satisfy a Cordes condition, we will show the design and analysis of stable finite element methods of arbitrary order, with quasi-optimal approximation bounds and optimal rates of convergence. We will finally present recent work on our proof of convergence of the method when the mesh refinements are determined adaptively by a posteriori error estimators.

This talk is based on joint work with E. Süli (Oxford) and E. L. Kawecki (formerly UCL).

Denis Talay (Ecole Polytechnique)

Title: Convergence to diffusion invariant measures: a survey of old techniques and related open questions

Abstract: In this talk we present a survey of techniques to analyze the long time behaviour of parabolic PDEs, diffusion processes and discretization schemes for diffusion processes. We also present a few related open questions.