Workshop in Probability and Stochastic Analysis

– Schedule –

Lecture Room 4, Mathematical Institute, University of Oxford

Thursday, 25th of April

14:00 – 14:50	Xue-mei Li (Imperial College)	A new stochastic variation
14:50 – 15:40	Hakima Bessaih (University of Wyoming)	Numerical schemes for the 2d Stochastic Navier-Stokes equations: Strong rate of convergence
15:40 – 16:15	**Coffee Break**	
16:15 – 17:05	Francesco Russo (ENSTA ParisTech)	BSDEs, martingale problems, associated deterministic equations and applications to hedging under basis risk

A new stochastic variation

Xue-mei Li
Imperial College

A perturbation to the Brownian motion was applied by Bismut for the first variation on the canonical horizontal stochastic differential equation on the orthonormal frame bundle. This was later used to give an integration by part formula, by Driver among others. The same perturbation was used by Stroock and Malliavin obtaining gradient and Hessian estimates of the logarithmic heat kernel, these methods do not appear to work for non-compact manifolds for two reasons. Firstly the first exit time from a domain behaves badly with respect to the initial position. In this joint work with X. Chen and B. Wu we use a different perturbation to obtain a desired formula. The proof is clean (however involving covariant differentiation of the structure equations on the frame bundle, not needed for the first variation.) We will also construct a family of adapted random cut-off processes with local uniform moment bounds, and use them for obtaining comparison theorems for the gradient and the Hessian. Our results are valid fro any a general complete Riemannian manifolds without curvature restrictions.

Numerical schemes for the 2d Stochastic Navier-Stokes equations: Strong rate of convergence.

Hakima Bessaih
University of Wyoming

We consider a time discretization scheme of Euler type for the 2d stochastic Navier-Stokes equations on the torus. We prove a mean square rate of convergence. This refines previous results established with a rate of convergence in probability only. Using exponential moment estimates of the solution of the Navier-Stokes equations and a convergence of a localized scheme, we can prove strong convergence of fully implicit and semi-implicit time Euler discretization and also a splitting scheme. The speed of convergence depends on the diffusion coefficient and the viscosity parameter.

If time permits, an introduction to some anisotropic 3d models will be given with their numerical schemes.

BSDEs, martingale problems, associated deterministic equations and applications to hedging under basis risk.

Francesco Russo
ENSTA ParisTech, http://uma.ensta-paristech.fr/∼russo

The talk will be based on partial joint work with Adrien Barrasso (ENSTA ParisTech) and Ismail Laachir (ZELIADE) .

The aim of this talk consists in introducing a new formalism for the deterministic analysis associated with backward stochastic differential equations driven by general càdlàg martingales, coupled with a forward process.

When the martingale is a standard Brownian motion, the natural deterministic analysis is provided by the solution u of a semilinear PDE of parabolic type coupled with a function v which is associated with the ∇u, when u is of class C^1 in space. When u is only a viscosity solution of the PDE, the link associating v to u is not completely clear: sometimes in the literature it is called the *identification* problem.

The idea is to introduce a suitable analysis to investigate the equivalent of the identification problem in a general Markovian setting with a class of examples. An interesting application concerns the hedging problem under basis risk of a contingent claim $g(X_T, S_T)$, where S (resp. X) is an underlying price of a traded (resp. non-traded but observable) asset, via the celebrated Föllmer-Schweizer decomposition. We revisit the case when the couple of price processes (X, S) is a diffusion and we provide explicit expressions when (X, S) is an exponential of additive processes. Extensions to non-Markovian (path-dependent cases) are discussed.