Durham University

Workshop on Stochastic Analysis and Ergodic Theory

coordinated by the Bielefeld-Durham-Edinburgh-Swansea Stochastic Analysis Network

Sponsored by the EPSRC and Royal Society Newton Fund

Local organisers: *Chunrong Feng and Huaizhong Zhao*

Department of Mathematical Sciences, Durham University
Venue: MCS2068, 16-18 December 2024

Speakers:

Theo Assiotis (Edinburgh)
Rui Bai (Durham)
Zdzisław Brzeźniak (York)
Ofer Busani (Edinburgh)
Chunrong Feng (Durham)
Dmitri Finkelshtein (Swansea)
Sebastian Grube (Bielefeld)
Istvan Gyongy (Edinburgh)
Zimo Hao (Bielefeld)
Jiawei Li (Edinburgh)
Aristide Ngana (York)
Sotirios Sabanis (Edinburgh)
Michael Röckner (Bielefeld)
Leonardo Tolomeo (Edinburgh)
Oliver Tough (Durham)
Huaizhong Zhao (Durham)
Xianliang Zhao (Bielefeld)

UKRI Engineering and Physical Sciences Research Council

THE ROYAL SOCIETY

Workshop on Stochastic Analysis and Ergodic Theory
Department of Mathematical Sciences, Durham University

Venue: MCS 2068

Schedule of talks

Monday 16 December 2024

15:00-15:50 **Michael Röckner (Bielefeld),** *p–Brownian motion and the p–Laplacian*

15:50-16:10 Tea/Coffee

16:10-17:00 **Chunrong Feng (Durham),** *Ergodicity of non-stationary stochastic processes*

17:00-17:50 **Sotirios Sabanis (Edinburgh**), *From Langevin based optimizers to diffusion based generative models*

Tuesday 17 December

9:30-10:20 **Zdzisław Brzeźniak (York),** *Stochastic PDEs with constraints*

10:20-11:10 **Sebastian Grube (Bielefeld),** *Strong solutions to degenerate SDEs and uniqueness for degenerate Fokker–Planck equations*

11:10-11:30 Tea/Coffee

11:30-12:20 **Huaizhong Zhao (Durham),** *Ergodic theory under nonadditive probabilities/sublinear expectations*

Lunch

14:00-14:50 **Ofer Busani (Edinburgh),** *Decorrelation of the KPZ fixed point from the flat initial condition*

14:50-15:40 **Zimo Hao (Bielefeld),** *Singular kinetic McKean-Vlasov SDEs*

15:40-16:00 Tea/Coffee

16:00-16:50 **Jiawei Li (Edinburgh),** *Pathwise well-posedness of stochastic nonlinear wave equations with multiplicative noise*

16:50-17:40 **Rui Bai (Durham),** *Large deviations principle for invariant measures of stochastic Burgers equations*

Wednesday 18 December

9:30-10:20 **Leonardo Tolomeo (Edinburgh),** *Statistical mechanics of the focusing nonlinear Schrödinger equation*

10:20-11:10 **Aristide Ngana (York),** *Existence of global weak solutions of 3D ferrohydrodynamic equations with transport noise I: Bloch-Torrey regularisation*

11:10-11:30 Tea/Coffee

11:30-12:20 **Dmitri Finkelshtein (Swansea),** *Spatio-temporal correlations in a class of interacting particle systems*

Lunch

14:00-14:50 **Oliver Tough (Durham),** *Longtime behaviour of the stochastic FKPP equation conditioned on non-fixation*

14:50-15:40 **Theo Assiotis (Edinburgh),** *Infinite-dimensional diffusions from random matrix dynamics*

15:40-16:30 **Xianliang Zhao (Bielefeld),** *Graphon particle systems with singular kernels:mean-field limits and propagation of chaos*

Title and Abstract of talks

Theo Assiotis (Edinburgh)

Title: *Infinite-dimensional diffusions from random matrix dynamics*

Abstract: I will talk about the infinite particle limit of eigenvalue stochastic dynamics introduced by Rider and Valko. These are the canonical dynamics associated to the inverse Laguerre ensemble in the way Dyson Brownian motion is related to the Gaussian ensemble. For this model we can prove convergence, from all initial conditions, to a new infinite-dimensional Feller process, describe the limiting dynamics in terms of an infinite system of log-interacting SDE that is out-of-equilibrium and finally show convergence in the long-time limit to the equilibrium state given by the (inverse points of the) Bessel determinantal point process.

Rui Bai (Durham)

Title: *Large deviations principle for invariant measures of stochastic Burgers equations*

Abstract: We study the small noise asymptotic for stochastic Burgers equations on $(0,1)$ with the Dirichlet boundary condition. We consider the case in which the noise is more singular than space-time white noise. We let the noise magnitude $\sqrt{\epsilon} \rightarrow 0$ and the covariance operator Q_ϵ converge to $(-\Delta)^{\frac 1 2}$ and prove the large deviations principle for solutions, uniformly with respect to the bounded initial value of the equation. Furthermore, we set Q_ϵ to be a trace class operator and converge to $(-\Delta)^{\frac{\alpha}{2}}$ with $\alpha<1$ in a suitable way such that the invariant measures exist. Then, we prove the large deviations principle for the invariant measures of stochastic Burgers equations.

Zdzisław Brzeźniak (York)

Title: *Stochastic PDEs with constraints*

Abstract: We will consider deterministic and stochastic nonlinear heat equation subject to the constraint that the L^2-norm is preserved. The problem is well posed in the space $H^1\cap L^p$ for an appropriate p. We consider p large so that $H^1\notsubset L^p$. The large deviations principle will also be discussed.

Ofer Busani (Edinburgh)

Title: *Decorrelation of the KPZ fixed point from the flat initial condition*

Abstract: The KPZ class is a set of 1+1 random growth interface models that are believed to model the statistics of an interface of a growing two-dimensional surface. Upon a time-space scaling , it is believed (and was proven for a handful of models) that all such models should converge to a universal scaling limit called the KPZ-fixed point. We shall discuss spatial decorrelation for the KPZ-fixed point. In particular, when the KPZ-fixed point starts from a specific initial condition called 'flat', we obtain the first order term in the exponent of the decay. Joint work with Riddhipratim Basu and Patrik Ferrari.

Chunrong Feng (Durham)

Title: *Ergodicity of non-stationary stochastic processes*

Abstract: As we know, many important works in ergodic theory of stochastic dynamical systems have been obtained for invariant measures and aperiodic stationary processes. However, there exist non-stationary but ergodic processes. I will discuss the ergodic theory of three kind of non-stationary stochastic processes, which are random periodic processes, random quasi-periodic processes and stochastic processes under nonlinear expectation spaces. My talk will mainly concentrate on random periodic processes and periodic measures. This talk is based on some joint work with Y. J. Liu, Y. Liu, Y. J. Liu, Baoyou Qu, Huaizhong Zhao and Johnny Zhong.

Dmitri Finkelshtein (Swansea)

Title: *Spatio-temporal correlations in a class of interacting particle systems*

Abstract: For interacting particle systems in the continuum, the study of spatial pair correlations between particles at different spatial positions is well-established, typically using second-order (spatial) correlation functions which represent factorial spatial moments of the system's states. However, the investigation of spatio-temporal pair correlations between particles located at different spatial positions in different moments in time has received relatively little attention and lacks a comprehensive approach. We introduce a general method suitable for a broad class of non-diffusive interacting particle systems in the continuum. Our approach simplifies the analysis of spatio-temporal correlations by relating them to the study of spatial correlations in auxiliary multi-type systems. We demonstrate our approach for several population dynamics and consider their mean-field and beyond-mean-field behaviour. Finally, we validate our theoretical predictions by comparing them to the results obtained through computer simulations. Based on a joint paper with Otso Ovaskainen and Panu Somervuo.

Sebastian Grube (Bielefeld)

Title: *Strong solutions to degenerate SDEs and uniqueness for degenerate Fokker–Planck equations*

Abstract: We study the existence of probabilistically strong solutions for stochastic differential equations with unbounded and discontinuous coefficients. Here, the diffusion coefficient is allowed to degenerate. Furthermore, we obtain new uniqueness results for the corresponding Fokker-Planck equations.

Zimo Hao (Bielefeld)

Title: *Singular kinetic McKean-Vlasov SDEs*

Abstract: In this talk, we delve into mean-field kinetic stochastic differential equations (SDEs) featuring singular interaction kernels driven by alpha-stable processes. We obtain the well-posedness and provide quantitative estimates for the propagation of chaos of moderately interacting N-particle systems related to the kinetic SDEs with singular interaction kernels, such as the Coulomb potential. The talk is based on joint works with Jean-Francois Jabir, Stephane Menozzi, Michael Röckner and Xicheng Zhang.

Jiawei Li (Edinburgh)

Title: *Pathwise well-posedness of stochastic nonlinear wave equations with multiplicative noise*

Abstract: Over the last decade, there has been a significant development in the study of stochastic dispersive PDEs, broadly interpreted with random initial data and/or additive stochastic forcing, where the difficulty comes from roughness in spatial regularity. In this talk, we consider the well-posedness of stochastic nonlinear wave equations with multiplicative noises, whose Ito solutions were constructed in 80's. The results on pathwise well-posedness for stochastic nonlinear wave equations (SNLW) will be presented. The main challenge of this problem comes from the deficiency of temporal regularities. We overcome this issue by bridging controlled rough paths and the Fourier restriction norm method. This talk is based on the joint work with Andreia Chapouto and Tadahiro Oh.

Aristide Ngana (York)

Title: *Existence of global weak solutions of 3D ferrohydrodynamic equations with transport noise I: Bloch-Torrey regularisation*

Abstract: In this talk, we present an electrically conductive ferrohydrodynamic system which describes the Bloch-Torrey regularization of the motion of an electrically conducting ferrofluid driven by transport noise filling a 3D bounded domain with a smooth boundary. The system under study is basically a coupling of the Navier-Stokes equations with internal rotation, the Maxwell equations and the ferromagnetisation equations. Our main focus is to establish the global existence of a probabilistic weak solution for this stochastic system, assuming the electrical conductivity satisfies specific smallness conditions.

Michael Röckner (Bielefeld)

Title: *p–Brownian motion and the p–Laplacian*

Abstract: In this talk we shall present the construction of a stochastic process, which is related to the parabolic p-Laplace equation in the same way as Brownian motion is to the classical heat equation given by the (2-) Laplacian.

Joint work with:

1) Viorel Barbu, Al.I. Cuza University and Octav Mayer Institute of Mathematics of Romanian Academy, Ia.si, Romania

2) Marco Rehmeier, Faculty of Mathematics, Bielefeld University, Germany

References

[1] V. Barbu, M. Rehmeier, M. RÅNockner: arXiv:2409.18744
[2] V. Barbu, M. RÅNockner: Springer LN in Math. 2024

Sotirios Sabanis (Edinburgh)

Title: *From Langevin based optimizers to diffusion based generative models*

Abstract: We will review some recent advances in the field of Langevin based optimizers with emphasis on their application in training neural nets. We will highlight interesting links between this class of stochastic optimizers and diffusion-based (known also as scored-based) generative models.

Leonardo Tolomeo (Edinburgh)

Title*: Statistical mechanics of the focusing nonlinear Schrödinger equation*

Abstract: In this talk, we discuss a number of results related to (non-)construction of the Gibbs measures for focusing nonlinear Schrödinger equations. The program was initiated by Lebowitz-Rose-Speer (1988), who built the focusing Φ^p measure in dimension 1 by introducing a suitable mass cutoff. A number of open questions were raised in this work, namely the existence of certain phase transitions and if the construction can be repeated in higher dimension. In this talk, we will answer most of those questions. After surveying various results, we will discuss the general strategy for tackling these problems, and the main techniques that had been developed to find their solution.

This talk is based on joint works with T. Oh (Edinburgh), M. Okamoto (Osaka), H. Weber (Münster) and J. Forlano (Edinburgh).

Oliver Tough (Durham)

Title: *Longtime behaviour of the stochastic FKPP equation conditioned on non-fixation*

Abstract: The stochastic FKPP equation is a stochastic PDE which provides a prototypical model for the evolution of the spatial distribution of a given gene type in a large population under the effects of migration, natural selection and genetic drift. It undergoes fixation, representing the given gene type or its complement disappearing from the population. We prove that the stochastic FKPP on the circle killed upon fixation has a unique quasi-stationary distribution, and that the distribution conditioned on non-fixation converges to this unique QSD for any initial condition. Moreover we obtain the asymptotics of the fixation time, as a function of the initial condition. Whereas there is a large literature addressing such questions for finite-dimensional processes, this is one of the very first works to do so in the SPDE or infinite dimensional setting, and the first for a physically relevant SPDE model.

Huaizhong Zhao (Durham)

Title: *Ergodic theory under nonadditive probabilities/sublinear expectations*

Abstract: In this talk, I will discuss the ideas in the development of ergodic theory under the sublinear expectation space and its analogue upper probabilities. In mind with irreducibility as its key essence, ergodicity is defined as that any invariant set has upper probability 0 or its complement upper probability 0 (Feng-Zhao (SIMA 2021/ Preprint 2017 arXiv:1705.03549)). The ergodicity is equivalent to the irreducibility of the measurable dynamical systems, the eigenvalue 1 of the Koopman operator being simple, Birkhoff's law of large numbers with

single value. Under sublinear Markov setup, the theory was also developed via a corresponding lifting canonical dynamical system. It is also proved that the G-Brownian motion on the unit circle is ergodic as an example.

Following this initial work, more recently a number of progresses have been made including Feng-Wu-Zhao (SPA 2020) on capacity, Ma-Zhao (Preprint 2024) on ergodic controls, Zhao-Zhao (Preprint 2024) on ergodicity of G-diffusions, and Feng-Huang-Liu-Zhao (Preprints 2023, 2024 arXiv:2411.00663; arXiv:2411.02030) described below. In this talk I can only concentrate a few results in the latter. First, under the upper probability preserving system (UPPS) set up, the ergodicity is equivalent to that an invariant skeleton measure exists, is unique and ergodic. The invariant skeleton then gives the precise formula of space average as the limit of time averaging in the Birkhoff type law of large numbers. It also characterizes ergodicity with weak independence. Moreover, with the equivalent condition on ergodicity, we define weakly mixing as when the eigenvalue 1 is unique and give equivalent conditions such as ergodicity on product space and asymptotic independence. I will also discuss a weaker regime that any invariant set has upper probability 0 or 1, in parallel to an ergodicity definition of a classical probability, proposed by Cerreia-Vioglio, Maccheroni and Marinacci (2016). We found that this does not give the irreducibility, however, is equivalent to UPPS being of finite ergodic components, Birkhoff's ergodic theorem with finite multiple values and the eigenvalue 1 of the Koopman operator being of finite multiplicity.

Xianliang Zhao (Bielefeld)

Title*: Graphon particle systems with singular kernels:mean-field limits and propagation of chaos*

Abstract: This talk focuses on the study of interacting diffusion processes on dense graphs with singular kernels. We will discuss key results, including the derivation of mean-field limits, the propagation of independence, and—under specific restrictive conditions on the limiting graphon—the propagation of chaos. The analysis relies on a tightness argument, the Fisher information framework for both particle systems and the limiting McKean-Vlasov equation, the measurable maximal theorem, and the theory of graph limits.