

Let  $D = (0, 1)$ .

- a) Provide a sequence of functions  $f_n : D \rightarrow \mathbb{R}$  that converges **uniformly** to some function  $f : D \rightarrow \mathbb{R}$  on  $D$ . *Justify your answer.*
- b) Provide a sequence of functions  $f_n : D \rightarrow \mathbb{R}$  that converges **pointwise but not uniformly** to some function  $f : D \rightarrow \mathbb{R}$  on  $D$ . *Justify your answer.*
- c) Provide a sequence of functions  $f_n : D \rightarrow \mathbb{R}$  that converges **uniformly on every compact subinterval of  $D$** , but not on the full interval  $D$ . *Explain how this function meets both conditions.*
- d) In what sense is uniform convergence on compact subintervals the most appropriate notion of convergence on the open interval  $D$ ?