36. (*). A bag contains three red and two white marbles. These are randomly laid out in a row.

(a) Write down the list of all outcomes, each of which is equally likely. Count by hand the total number of outcomes.

(b) Count by hand the number of outcomes in your list for which the white marbles appear side by side.

(c) From these two counts, derive the probability that the two white marbles appear side by side.

42. (*)**.** An n card hand is simply a selection of n out of 52 cards from a deck, selected without replacement, whose ordering is irrelevant.

(a) How many three card hands are there?

(b) How many three card hands are there, where each card is a face card (Jack, Queen, or King)?

(c) What is the probability of a three card hand where each card is a face card?

45. (**)**.** In the national lottery, you identify 6 balls from a set of 49 balls. Then 6 balls are selected at random, without replacement, from the 49 balls and you get a prize if at least 3 of these match the balls you identified, regardless of the order. (We ignore the bonus ball for now.)

What is the probability that you get a prize of some sort? You will need a calculator.

[Hint: How many outcomes match exactly 3 of the balls you selected? How many outcomes match exactly 4 of the balls you selected?]

48. (**)**.** A small village has population 30, of whom 20 support the Red party and 10 support the Blue party. An opinion pollster samples precisely 10 villagers at random. Find the probability that precisely 7 of the 10 villagers in the sample support the Red party. You will need a calculator.

25. (*). There is a probability of $3/4$ that Eric goes to the pub on Sunday evening (event A), and there is a probability of $1/3$ that Eric arrives late to work on Monday morning (event B). Without making any assumptions on the dependence of these two events, show that

$$\frac{1}{12} \leq \mathbb{P}(A \cap B) \leq \frac{1}{3}.$$

29. (*). Consider the sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$. Which of the following are σ-algebras, and which are not? Justify your answers.

(i) $\mathcal{F}_1 = \{\emptyset, \{1, 2, 3\}, \{4, 5, 6\}, \Omega\}$;

(ii) $\mathcal{F}_2 = \{\emptyset, \{1, 2, 3\}, \{4, 5\}, \{6\}, \Omega\}$;

(iii) $\mathcal{F}_3 = \{\emptyset, \{1, 2, 3, 4\}, \{1, 2, 3, 4, 5\}, \{1, 2, 3, 4, 6\}, \{5\}, \{6\}, \{5, 6\}, \Omega\}$.

50. (***). Seven standard cubic dice are thrown.

(a) What is the probability that all six numbers show up on the dice? [You will need a calculator.]

(b) What is the probability that exactly five numbers show up on the dice? [You will need a calculator.]